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STUDENTS' ERRORS IN SOLVING CALCULUS PROBLEMS ON INCREASING, DECREASING, AND CONCAVITY OF FUNCTIONS

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ABSTRACT
Calculus is a fundamental course in mathematics education; however, many students experience difficulties that lead to errors in problem solving. This study aimed to identify and analyze students' errors in solving calculus problems involving increasing functions, decreasing functions, and function concavity. A qualitative descriptive case study approach was employed involving 22 fifth-semester mathematics education students at UIN Sheikh Ali Hasan Ahmad Addary Padangsidimpuan during the 2024/2025 academic year. Data were collected through four essay-test items and semi-structured interviews with six purposively selected students representing different error categories. The data were analyzed using the interactive model of Miles, Huberman, and Saldaña, including data condensation, data display, and conclusion drawing. The findings revealed three categories of errors: conceptual, operational, and factual errors. Operational errors were the most dominant, particularly in derivative calculations, algebraic manipulations, and interval determination. Conceptual errors occurred when students misinterpreted the meaning of first and second derivatives in determining function behavior, whereas factual errors were related to the incorrect use of mathematical symbols and notation. The errors were mainly influenced by insufficient conceptual understanding, lack of carefulness, and ineffective problem-solving practices. These findings provide important implications for calculus instruction by highlighting the need to strengthen conceptual understanding alongside procedural fluency. The study contributes to the literature on calculus error analysis by providing an in-depth examination of students' difficulties in applying derivative concepts to analyze increasing functions, decreasing functions, and function concavity.
Keywords: calculus; , conceptual errors; error analysis,; function concavity, operational errors.

ABSTRAK
Kalkulus merupakan salah satu mata kuliah dasar dalam pendidikan matematika. Namun, banyak mahasiswa mengalami kesulitan yang menyebabkan terjadinya kesalahan dalam menyelesaikan masalah matematika. Penelitian ini bertujuan untuk mengidentifikasi dan menganalisis kesalahan mahasiswa dalam menyelesaikan soal kalkulus yang berkaitan dengan fungsi naik, fungsi turun, dan kecekungan fungsi. Penelitian ini menggunakan pendekatan kualitatif dengan desain studi kasus deskriptif yang melibatkan 22 mahasiswa semester V Program Studi Pendidikan Matematika di UIN Syekh Ali Hasan Ahmad Addary Padangsidimpuan pada tahun akademik 2024/2025. Data dikumpulkan melalui empat butir soal uraian dan wawancara semi-terstruktur terhadap enam mahasiswa yang dipilih secara purposive untuk mewakili kategori kesalahan yang berbeda. Data dianalisis menggunakan model analisis interaktif Miles, Huberman, dan Saldaña yang meliputi kondensasi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan adanya tiga kategori kesalahan, yaitu kesalahan konseptual, kesalahan operasional, dan kesalahan faktual. Kesalahan operasional merupakan jenis kesalahan yang paling dominan, terutama dalam perhitungan turunan, manipulasi aljabar, dan penentuan interval. Kesalahan konseptual terjadi ketika mahasiswa salah menafsirkan makna turunan pertama dan turunan kedua dalam menentukan perilaku fungsi, sedangkan kesalahan faktual berkaitan dengan penggunaan simbol dan notasi matematika yang tidak

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tepat. Kesalahan-kesalahan tersebut terutama dipengaruhi oleh kurangnya pemahaman konseptual, rendahnya ketelitian, serta praktik pemecahan masalah yang kurang efektif. Temuan penelitian ini memberikan implikasi penting bagi pembelajaran kalkulus dengan menekankan perlunya penguatan pemahaman konseptual yang seimbang dengan kelancaran prosedural. Penelitian ini berkontribusi pada kajian analisis kesalahan dalam kalkulus melalui penyajian analisis mendalam mengenai kesulitan mahasiswa dalam menerapkan konsep turunan untuk menganalisis fungsi naik, fungsi turun, dan kecekungan fungsi.
Kata Kunci: *analisis kesalahan, kalkulus, kesalahan konseptual, kecekungan fungsi, kesalahan operasional.*

Introduction

Mathematics is a fundamental discipline that plays a crucial role in developing logical, analytical, and systematic thinking skills and serves as a foundation for scientific and technological advancement. Despite its importance, mathematics remains one of the most challenging subjects for many students because it requires abstract reasoning, conceptual understanding, and procedural fluency. As a result, students often experience difficulties in understanding mathematical concepts and solving mathematical problems effectively (Kurniawan et al., 2025; Lastari et al., 2025; Sadiyah et al., 2025). In higher education, mathematics is an essential component of mathematics education programs, where students are expected not only to master mathematical concepts but also to develop pedagogical competencies as future mathematics teachers (Intan et al., 2022). Therefore, a strong conceptual understanding of mathematics is necessary to support both academic success and professional preparation.

One of the core subjects in mathematics education programs is calculus. Calculus studies change and accumulation through concepts of differentiation and integration and has extensive applications in mathematics, science, engineering, economics, and technology (Aulia et al., 2026; Nisa' et al., 2026; Renaya Dwi Septiani & Harisman, 2025). Beyond its practical applications, calculus develops students' analytical reasoning, problem-solving abilities, and mathematical thinking. However, calculus is widely recognized as a difficult subject because it requires students to integrate conceptual understanding, algebraic manipulation, graphical interpretation, and procedural knowledge simultaneously. Consequently, many students encounter difficulties that lead to various forms of errors when solving calculus problems.

Previous studies have consistently reported that students make numerous errors in calculus, particularly in differentiation and integration topics. Hajerina et al. and Anggoro, for example, found that students frequently experience conceptual misunderstandings, procedural mistakes, computational errors, and notation-related errors (Hajerina et al., 2022; Yudhi Anggoro, 2023a). Similarly, Andriyanti et al., Suhendra et al., and Wahyuni et al. revealed that students' errors are often influenced by insufficient conceptual understanding, inattention to problem requirements, and lack of procedural accuracy (Andriyanti et al., 2025; Suhendra et al., 2025; Wahyuni et al., 2025). These findings indicate that student errors in calculus are multidimensional and cannot be explained solely by computational weaknesses.

Although previous studies have provided valuable insights into students' difficulties in calculus, most of them focus on general differentiation or integration topics. Consequently, the specific characteristics of students' errors in solving problems involving increasing functions, decreasing functions, and function concavity remain underexplored. This limitation is important because these topics represent advanced applications of derivatives that require students to connect derivative concepts with function behavior, graphical interpretation, interval analysis, and mathematical reasoning (Tall, 2008). Unlike routine differentiation tasks, determining intervals of increase and decrease and analyzing function concavity require students to interpret the meaning of first and second derivatives conceptually rather than merely applying differentiation procedures.

For prospective mathematics teachers, mastery of increasing functions, decreasing functions, and function concavity is particularly important because these concepts form the foundation for understanding function analysis, optimization problems, graph sketching, and mathematical modeling. Moreover, future mathematics teachers are expected not only to solve such problems correctly but also to explain the underlying concepts and reasoning processes to their future students (Hussein, 2022). Students who have visual, auditory and kinesthetic learning styles have different ways of solving problems (Ningrum et al., 2025). Difficulties in understanding these concepts may therefore affect both their mathematical competence and their future teaching effectiveness.

To provide a stronger basis for analyzing students' errors, this study adopts the perspective of error analysis and cognitive learning theory. Error analysis views students' mistakes as valuable sources of information for understanding learning difficulties and identifying misconceptions (Hiebert, 2013). In this study, students' errors are classified into three categories: conceptual errors, operational errors, and factual errors. Conceptual errors refer to students' inability to understand or apply mathematical concepts correctly, such as misunderstanding the meaning of the first derivative in determining increasing and decreasing intervals or the second derivative in determining concavity. Operational errors refer to mistakes in performing mathematical procedures or calculations despite understanding the underlying concepts. Factual errors involve incorrect use of mathematical symbols, notation, definitions, or mathematical facts. These categories provide a systematic framework for identifying the nature of students' difficulties and interpreting the factors that contribute to their occurrence.

The theoretical foundation of this study is also informed by cognitive learning theory, which emphasizes that learning difficulties arise when students fail to construct meaningful relationships between existing knowledge and newly acquired concepts (Piaget, 1977). In calculus learning, students often memorize derivative rules procedurally without developing a deep conceptual understanding of how derivatives describe function behavior. As a result, students may successfully calculate derivatives but still struggle to interpret their meaning in determining intervals of increase, decrease, and concavity.

A critical review of previous studies also reveals several unresolved issues. While some studies emphasize conceptual errors as the dominant source of

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students' difficulties, others report that procedural or computational errors occur more frequently. Furthermore, most studies focus on identifying the existence of errors without exploring in depth how these errors emerge in specific calculus topics and what cognitive factors contribute to their occurrence. Therefore, a more focused investigation is needed to examine students' errors in topics that require both conceptual interpretation and procedural competence.

In addition to identifying the types of errors, understanding the underlying causes of students' errors is equally important because similar errors may arise from different cognitive processes. Previous studies have suggested that students' mathematical errors are not merely the result of computational weaknesses but are often associated with deeper cognitive and affective factors. Conceptual misunderstandings frequently occur when students possess incomplete or fragmented knowledge structures, preventing them from connecting derivative procedures with their conceptual meanings (Hajerina et al., 2022; Yudhi Anggoro, 2023a). Furthermore, cognitive learning theory suggests that meaningful learning occurs when new concepts are successfully integrated into existing cognitive schemas. When such integration fails, students tend to rely on memorized procedures without understanding the underlying concepts, leading to conceptual and procedural errors.

In addition to conceptual difficulties, several empirical studies have reported that carelessness, lack of attention to problem requirements, and hurried problem-solving behavior contribute significantly to students' mathematical errors (Andriyanti et al., 2025; Wahyuni et al., 2025). Students often overlook important information, misinterpret mathematical symbols, or perform calculations inaccurately despite possessing adequate conceptual knowledge. Such errors may be associated with limitations in metacognitive regulation, including inadequate monitoring, checking, and evaluation of solution processes. Moreover, time pressure, low confidence, and anxiety may further increase the likelihood of making avoidable mistakes. Therefore, students' errors should be viewed as complex phenomena influenced by interactions among conceptual understanding, procedural knowledge, metacognitive abilities, and affective factors rather than as isolated computational mistakes.

Given the complexity of these factors, a qualitative approach is considered particularly appropriate for this study. While quantitative methods can measure the frequency and distribution of errors, they often provide limited insight into the reasoning processes that lead students to make those errors. In contrast, qualitative methods enable researchers to explore students' thinking processes, interpretations, and decision-making strategies in greater depth. Through written responses and semi-structured interviews, researchers can investigate not only what errors students make but also why those errors occur and how students understand the concepts involved. Interviews are especially valuable because they allow students to explain their reasoning, reveal misconceptions, clarify ambiguous written responses, and describe the difficulties they experience during problem solving. Consequently, a qualitative approach provides richer and more comprehensive information regarding students' conceptual understanding and the cognitive factors underlying their errors than could be obtained solely through quantitative analysis (Creswell & Poth, 2018).

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Therefore, employing a qualitative design allows this study to generate a deeper understanding of mathematics education students' errors when solving problems involving increasing functions, decreasing functions, and function concavity. Such understanding is essential for developing instructional interventions that address not only the observable errors but also the cognitive and conceptual difficulties that produce them.

Based on this review, a clear research gap can be identified. Existing studies have not systematically analyzed students' conceptual, operational, and factual errors in solving problems related to increasing functions, decreasing functions, and function concavity, nor have they comprehensively explored the factors contributing to these errors through qualitative investigation. Consequently, limited information is available regarding how mathematics education students understand and apply derivative concepts when analyzing function behavior.

The novelty of this study lies in its comprehensive qualitative analysis of students' errors in three closely related but conceptually demanding calculus topics—increasing functions, decreasing functions, and function concavity—using the framework of conceptual, operational, and factual errors supported by interview data. Unlike previous studies that primarily examine differentiation in general, this study focuses specifically on derivative applications that require conceptual interpretation and mathematical reasoning. In addition, this study investigates not only the types of errors but also the underlying causes of those errors from students' perspectives.

The findings of this study are expected to contribute both theoretically and practically. Theoretically, the study enriches the literature on calculus learning and error analysis by providing a deeper understanding of students' cognitive difficulties in interpreting derivative concepts. Practically, the findings can assist lecturers in designing more effective instructional strategies, developing targeted remedial interventions, and improving calculus curricula in mathematics education programs. Furthermore, the results may support the preparation of future mathematics teachers who possess stronger conceptual understanding and greater accuracy in solving and explaining calculus problems.

Therefore, this study aims to: (1) identify the types of errors made by mathematics education students in solving problems related to increasing functions, decreasing functions, and function concavity; (2) analyze the factors contributing to these errors; and (3) provide recommendations for improving calculus instruction and students' conceptual understanding of derivative applications.

Research Methods

Research Design

This study employed a qualitative descriptive case study design to investigate the types and causes of students' errors in solving calculus problems involving increasing functions, decreasing functions, and function concavity. A qualitative approach was selected because the purpose of the study was not merely to determine the frequency of errors but to understand how and why those errors occurred through an in-depth exploration of students' reasoning processes, conceptual understanding, and problem-solving strategies. Unlike quantitative approaches, which primarily measure the prevalence of errors, qualitative inquiry

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enables researchers to uncover the cognitive and contextual factors underlying students' mistakes through detailed analysis of written responses and interview data (Yin, 2018).

The study adopted a descriptive case study design because it focused on a bounded case, namely a single cohort of fifth-semester mathematics education students enrolled in a Calculus course at UIN Sheikh Ali Hasan Ahmad Addary Padangsidimpuan during the 2024/2025 academic year. The case was bounded by the specific learning context, course content, participants, and academic period. A case study design was considered appropriate because the research aimed to obtain a holistic understanding of students' errors within their natural learning environment rather than to generate a general theory (grounded theory), describe lived experiences (phenomenology), or construct personal narratives (narrative inquiry).

Research Setting and Participants

The study was conducted at UIN Syekh Ali Hasan Ahmad Addary Padangsidimpuan, Indonesia. Participants consisted of 22 fifth-semester students enrolled in the Mathematics Education Study Program who had completed instruction on derivative applications, including increasing functions, decreasing functions, and function concavity.

A total sampling technique was employed because the population consisted of only one class. Including all students allowed the researcher to capture the full range of error characteristics within the learning context and provided a comprehensive representation of students' difficulties in the selected calculus topics.

Development of Research Instruments

Two instruments were used: (1) an essay test and (2) a semi-structured interview guide.

The essay test was developed based on the learning outcomes of the Calculus course and the indicators of derivative application competencies. Initially, four essay items were developed to represent three calculus topics: increasing functions, decreasing functions, and function concavity. The items were designed to assess conceptual understanding, procedural competence, and the ability to interpret derivative information.

Table 1 Presents The Blueprint Of The Essay Test

Topic	Indicator	Cognitive Level	Error Category
Increasing functions	Determine intervals of increase using first derivatives	Analyze (C4)	Conceptual, operational
Decreasing functions	Determine intervals of decrease using first derivatives	Analyze (C4)	Conceptual, operational
Function concavity	Determine concavity using second derivatives	Analyze (C4)	Conceptual, operational
Combined application	Interpret function behavior from derivative information	Evaluate (C5)	Conceptual, operational, factual

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The use of multiple essay items was intended to ensure adequate representation of all targeted concepts and to provide richer evidence of students' error patterns.

Instrument Validation

Content validity was established through expert judgment involving three mathematics education experts. The experts evaluated the relevance of the content, clarity of instructions, appropriateness of difficulty level, and alignment with learning objectives.

The validity of each item was calculated using Aiken's V coefficient (Aiken, 1985). The obtained Aiken's V values ranged from 0.86 to 0.95, indicating high content validity. Suggestions provided by the validators were incorporated to improve wording clarity and mathematical notation before the instruments were administered.

The interview protocol was also reviewed by the experts and piloted with two students outside the research participants to ensure clarity and appropriateness of the questions.

Data Collection Procedures

Data collection was conducted in three stages.

Stage 1: Written Test

Students completed the essay test individually during regular class sessions. They were instructed to provide complete solution procedures to facilitate identification of conceptual, operational, and factual errors.

Stage 2: Semi-Structured Interviews

Based on the test results, six students were purposively selected for interviews. The selection represented different patterns of errors, including students who predominantly exhibited conceptual errors, operational errors, factual errors, and combinations of these categories.

The interview protocol consisted of ten open-ended questions designed to explore students' reasoning processes, conceptual understanding, strategies used during problem solving, and perceived causes of errors. Each interview lasted approximately 20–30 minutes and was audio-recorded with participants' consent. Interview recordings were subsequently transcribed verbatim for analysis.

Stage 3: Data Verification

Test responses and interview transcripts were compared to verify the consistency of identified errors and to explore the cognitive factors underlying students' mistakes.

Data Analysis

Data were analyzed using the interactive model of Miles, Huberman, and Saldaña, consisting of data condensation, data display, and conclusion drawing (Miles et al., 2014).

Data Condensation

Students' written responses and interview transcripts were reviewed repeatedly. Meaningful units related to students' errors were identified and assigned initial codes.

Examples of initial coding included:

Misinterpreting the meaning of the first derivative → Conceptual Error (CE)

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Incorrect derivative computation → Operational Error (OE)
Incorrect inequality symbols (<, >) → Factual Error (FE)

Data Display

The coded data were organized into matrices and summary tables to facilitate comparison across participants and error categories.

Conclusion Drawing and Verification

Patterns of errors and their underlying causes were identified through iterative comparison between test responses and interview data. Emerging interpretations were continuously verified against the original data to ensure consistency and accuracy.

Coding Validation and Trustworthiness

Several strategies were employed to ensure trustworthiness (Shenton, 2004).

Credibility was established through data triangulation between written tests and interviews, prolonged engagement with the data, and member checking, whereby participants reviewed interview summaries to confirm the accuracy of interpretations. Dependability was ensured through an audit trail documenting all stages of data collection and analysis.

Confirmability was maintained by preserving coding records, interview transcripts, and analytical memos to enable external review of the research process. Transferability was supported by providing detailed descriptions of participants, learning context, course content, and research procedures.

To minimize researcher bias, reflexive notes were maintained throughout the study to document assumptions, decisions, and interpretations developed during data analysis (Saldaña, 2021).

Frequency Analysis within the Qualitative Framework

Although this study employed a qualitative approach, simple frequency counts were used descriptively to indicate the occurrence of conceptual, operational, and factual errors. The frequencies were not intended for statistical inference but rather to support qualitative interpretation by identifying dominant patterns of errors among participants.

Ethical Considerations

Ethical approval for the study was obtained from the Faculty of Tarbiyah and Teacher Training, UIN Sheikh Ali Hasan Ahmad Addary Padangsidimpuan. Prior to participation, students received information regarding the purpose of the study and signed informed consent forms. Participation was voluntary, and participants were informed that they could withdraw at any stage without penalty.

To ensure confidentiality, pseudonyms were used in reporting findings, and all collected data were stored securely and used solely for research purposes.

Results and Discussion

The results obtained after administering the test to the students are as follows Table 1:

Table 1. Distribution of Students' Correct and Incorrect Answers on Calculus Problems

Question	Correct Answer	Percentage	Incorrect Answer	Percentage
1a	20	90.9%	2	9.1%
1b	19	86.4%	3	13.6%
2	17	77.3%	5	22.7%

Based on Table 1, most students were able to correctly solve problems related to increasing and decreasing functions. The highest percentage of correct answers was found in Question 1a (90.9%), while the lowest was found in Question 2 (77.3%), which focused on function concavity. These findings indicate that students generally possess a better understanding of first-derivative applications than second-derivative applications.

The lower performance on Question 2 suggests that determining function concavity requires more advanced conceptual reasoning because students must connect second-derivative procedures with graphical interpretations of function behavior. This finding highlights that function concavity remains a challenging topic even for mathematics education students who have already completed calculus instruction.

Based on the table above, for question 1a, 20 students (90.9%) answered correctly, and 2 students (9.1%) answered incorrectly. For question 1b, 19 students (86.4%) answered correctly, and 3 students (13.6%) answered incorrectly. For question 2, 17 students (77.3%) answered correctly, and 5 students (22.7%) answered incorrectly. The highest number of errors occurred in question 2, followed by questions 1a and 1b.

The errors identified in the students' incorrect answers include conceptual errors, operational errors, and factual errors, summarized in the following Table 2:

Table 2. Distribution of Conceptual, Operational, and Factual Errors

No.	Type of Error	Participant	Question
1	Conceptual	MS1	1a
		MS1	2
		MS4	2
2	Operational	MS10	1a
		MS16	1b
		MS17	1b
		MS2	2
		MS21	2
		MS22	2
3	Factual	MS20	1b

Based on Table 2, operational errors were the most frequently observed type of error among the participants. This result indicates that although students generally understood the underlying concepts, many experienced difficulties in accurately performing derivative calculations, simplifying algebraic expressions, and determining roots.

The dominance of operational errors suggests that procedural fluency remains a significant challenge in calculus learning. Such findings imply that students require not only conceptual understanding but also extensive practice in executing mathematical procedures accurately.

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From the table, it is apparent that operational errors were the most dominant type of error among students.

Question 1a

"Determine the intervals of increasing and decreasing functions for the following function: $f(x) = x^2 + 6x - 10$."

Students were required to determine the intervals by taking the first derivative of the function with respect to x and applying the concepts of increasing and decreasing functions. Two types of errors were observed In Figure 1.

Conceptual Error

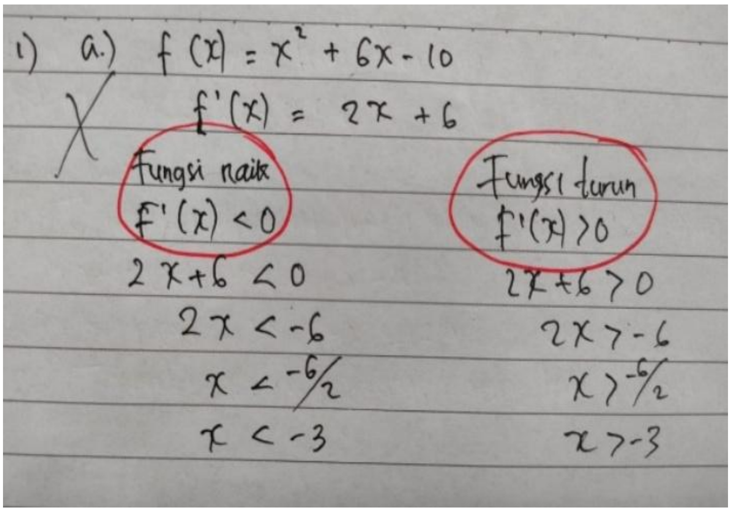

Figure 1. Conceptual Error Made by MS1 in Determining Increasing and Decreasing Functions

Figure 1 illustrates a conceptual error made by participant MS1. The student incorrectly interpreted the conditions for increasing and decreasing functions by reversing the derivative inequalities. Instead of recognizing that a function is increasing when $f'(x) > 0$ and decreasing when $f'(x) < 0$, the student applied the opposite interpretation.

This error indicates a misconception regarding the relationship between the sign of the first derivative and function behavior. Such misconceptions are particularly concerning because they may influence students' understanding of more advanced derivative applications, including optimization and graph sketching.

MS1's answer contained an error in determining the concept of increasing and decreasing functions, causing the signs to be reversed, where the increasing function was written as $f(x) < 0$ and the decreasing function as $f(x) > 0$. The following is an interview with MS1 regarding the conceptual error on question 1a.

Interviewer (P)	:	"Do you know what the error is on question 1a?"
MS1	:	"Yes, sir"
P	:	"What is the error on question 1a?"
MS1	:	"The signs of the increasing and decreasing functions, sir."
P	:	"On question 1a, you incorrectly positioned the concepts of increasing and decreasing functions. Why did this happen?"

MS1 : "I am not sure how to distinguish the concepts of increasing and decreasing functions, sir. I wrote increasing function as f(x) < 0 and decreasing function as f(x) > 0."

Based on this interview, it is evident that the student did not understand the difference between the concepts of increasing and decreasing functions, leading to an incorrect answer. Conceptual errors like this result in inaccurate outcomes in Figure 2.

Operational Error

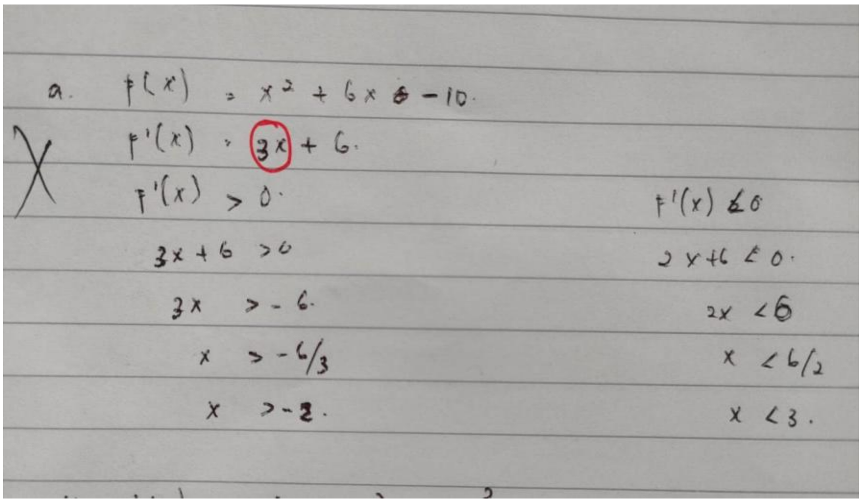

Figure 2. MS10's Answer

MS10's answer contained an operational error, specifically in the first derivative. The student calculated the first derivative as 3x + 6, while the correct answer was 2x – 6. This mistake in the derivative affected the final answer. The following is an interview with MS10 regarding the operational error on question 1a.

Interviewer (P)	:	"Look at question 1a. Where is the error?"
MS10	:	"Here, sir."
P	:	"This error is an operational error. Why did this happen?"
MS10	:	"Yes, sir. I was not careful when answering the question."
P	:	"Did you check your answer again?"
MS10	:	"No, sir."

Based on this interview, it is evident that the student lacked carefulness in completing the answer, which ultimately resulted in an incorrect response due to an operational mistake. Errors in computing the first derivative can lead to incorrect results for both increasing and decreasing functions.

Question 1b

"Determine the intervals of increasing and decreasing functions for: $f(x) = 3x^4 + 4x^3 - 12x^2$."

This question is similar to question 1a, with the only difference being the function itself. To determine the intervals of increasing and decreasing functions, the first derivative of the function is calculated first, followed by applying the concepts of increasing and decreasing functions in Figure 3.

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Operational Error

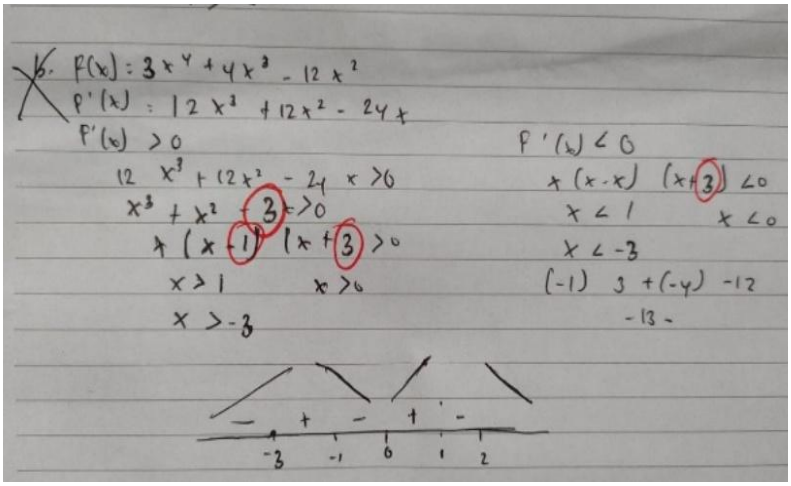

Figure 3. MS16's Answer

MS16's answer contained an operational error, resulting in an incorrect solution. The student made a mistake while simplifying the equation, writing $x^3 + x^2 - 3 > 0$ instead of the correct $x^3 + x^2 - 2 > 0$. Consequently, the result obtained was also incorrect. In addition, the student made errors in factoring the roots of the equation. The following is an interview with MS16 regarding the operational error on question 1b.

Interviewer (P)	:	"Look at this answer. What is wrong in this part?"
MS16	:	"I made an error in simplifying the equation, sir."
P	:	"So you made an operational error. Why did this happen?"
MS16	:	"I was somewhat rushed in completing it, sir."
P	:	"Besides that, you also made a mistake in factoring the roots. Didn't you check your answer after finishing?"
MS16	:	"No, sir. I submitted my answer immediately after finishing."

Based on this interview, it is evident that the student worked in a hurry, which prevented them from noticing the mistakes during simplification. Additionally, the student incorrectly factored the roots. Operational errors like these can cause the final answer to be incorrect in Figure 4.

Factual Error

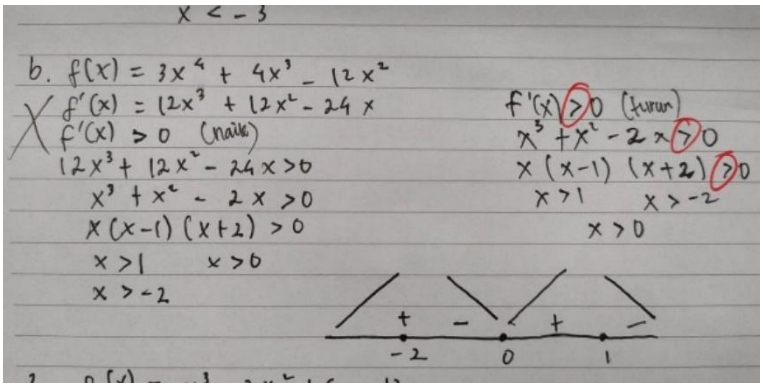

Figure 4. MS20's Answer

MS20's answer contained a factual error in the form of an incorrect directional sign ">". In the response, the student wrote ">" for the decreasing

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function, whereas the correct sign should have been "$<$", as a decreasing function is defined by $f'(x) < 0$. This mistake affected the final result for the decreasing function. The following is an interview with MS20 regarding the factual error on question 1b.

Interviewer (P) : "Look at this answer. Can you see where the mistake is?"
MS20 : "The sign, sir."
P : "Why did the sign mistake occur?"
MS20 : "Maybe because I was focused on looking at the increasing function, sir, so I didn't realize the sign direction was wrong."
P : "Alright, so do you know where the error is?"
MS20 : "Yes, sir. The sign direction is wrong."

Based on this interview, it is evident that the student was not careful when completing the question. The student did not realize that the sign direction they used was incorrect; the correct sign should have been "$<$".

Question 2

"Determine the intervals of concavity for the function $f(x) = x^3 - 3x^2 + 6x - 12$."

This question asks students to find the intervals of concavity for the given function. To determine these intervals, the function must first be differentiated twice, and then the solution is obtained by applying the concepts of concave upward and concave downward in Figure 5 and 6.

Conceptual Error

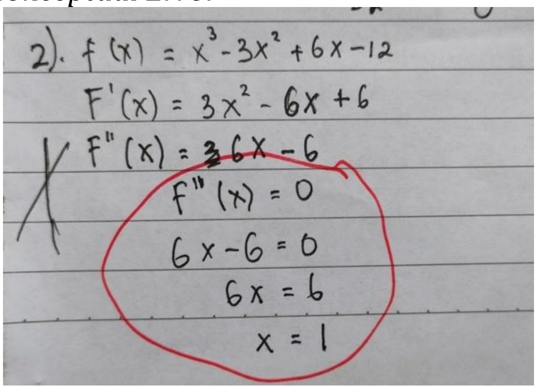

Figure 5. MS4's Answer

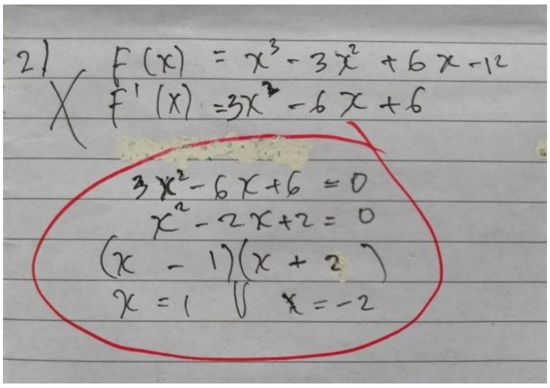

Figure 6. MS1's Answer

The answers of the two students showed differences but both fall under conceptual errors. MS4's answer (left) contained an error in which they set $F''(x) = 0$, knowing that $F''(x) = 0$ is used to find inflection points, whereas determining concavity requires considering two cases: concave upward ($f''(x) > 0$) and concave downward ($f''(x) < 0$). MS1's answer (right) contained an error in that they only computed the first derivative, while to determine concavity, the function must be differentiated twice (second derivative). Additionally, MS1 made an error similar to MS4 by setting $f' = 0$. These errors were caused by insufficient understanding of the concept of concavity, leading the students to confuse the concept of increasing and decreasing functions with concavity. The following is an interview with MS4 regarding the conceptual error on question 2.

Interviewer (P) : "On question 2, you made a mistake. Why is that?"

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MS4 : “I am confused, sir, about how to find the concavity of the function.”
P : “What concept did you apply?”
MS4 : “I don’t know, sir.”
P : “Alright. The concept you should use is similar to finding increasing and decreasing intervals, but for concavity, the function must be differentiated twice. If f’’(x) > 0, the function is concave upward; if f’’(x) < 0, it is concave downward. Do you understand now?”
MS4 : “Yes, sir.”

Based on this interview, it is evident that the student did not understand how to solve for function concavity, which led to an incorrect answer. As with previous conceptual errors, the final solution differs from the expected result. In Figure 7 and 8.

Operational Error

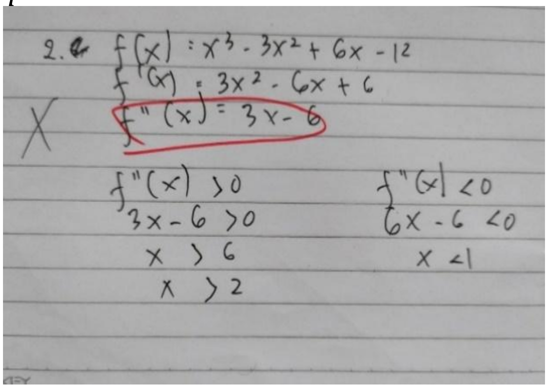

Figure 7. MS2’s Answer

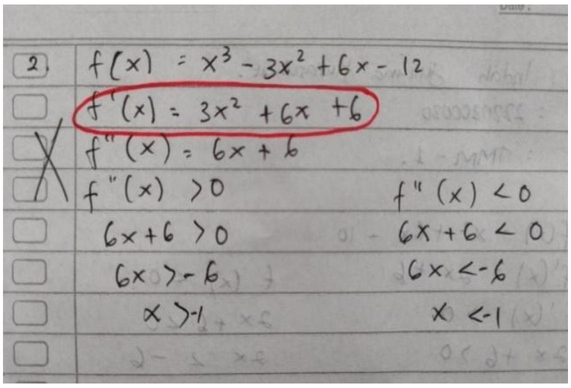

Figure 8. MS21’s Answer

The answers of the two students also contained errors, but both were of the same type: operational errors. In MS2’s answer (left), the error occurred in the second derivative, where the student wrote f’’(x) = 3x – 6 instead of the correct 6x – 6. MS21’s answer (right) contained an error in the first derivative, where the student wrote f’(x) = $3x^2$ + 6x + 6, while the correct answer was $3x^2$ – 6x + 6. MS21’s mistake was only in the sign of the 6x term, writing it as 6x instead of –6x. Mathematical operational errors can lead to incorrect student answers. The following is an interview with MS2 regarding the operational error on question 2:

Interviewer (P) : “Question 2 is incorrect. Do you know where the mistake is?”
MS2 : “Oh yes, sir. I know, sir.”
P : “Which part is wrong?”
MS2 : “The second derivative, sir, I wrote 3x – 6 instead of 6x – 6.”
P : “Why did your answer turn out wrong?”
MS2 : “I wasn’t careful, sir.”
P : “Since it was wrong at the beginning, it remained wrong to the end. Make sure to be careful next time, okay?”
MS2 : “Yes, sir.”

Based on this interview, it is evident that the student lacked carefulness in completing the answer. Operational errors, such as mistakes in computing

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derivatives, can lead to incorrect results, and sign errors in addition and subtraction can further affect the outcome of subsequent calculations.

Based on the results and discussion of this study, three types of student errors were identified in solving problems related to increasing functions, decreasing functions, and function concavity: (1) Conceptual errors: mistakes in applying the correct concept to the problem and failure to understand how to solve it correctly. (2) Operational errors: mistakes in computing first and second derivatives, errors in writing operational signs (addition and subtraction), and mistakes in factoring roots. (3) Factual errors: mistakes in placing symbols such as "<" and ">". According to the study conducted by (Yudhi Anggoro, 2023b), the types of student errors identified include mistakes in reading the question, misunderstanding the question, errors in process skills, and writing errors. Similarly, the study by (Manurung et al., 2024), reported that students make errors in solving problems, which can be categorized into three types: conceptual errors, procedural errors, and technical errors. The factors causing these errors are attributed to students' lack of carefulness, haste in completing the problems, and inability to correctly determine the problem-solving concepts.

The results of this study show that student errors in solving calculus problems related to increasing functions, decreasing functions, and function concavity can be divided into three main categories: conceptual errors, operational errors, and factual errors. Based on the test data, the most dominant errors were operational, observed in six students, followed by conceptual and factual errors.

Conceptual errors occur when students are unable to correctly apply the fundamental principles of increasing, decreasing, and concavity functions. For example, some students did not understand that an increasing function is determined by a positive first derivative, while a decreasing function is determined by a negative first derivative, resulting in incorrectly determined function intervals. This finding is consistent with Hajerina, Suciati, & Mailili , who stated that conceptual errors arise from an immature understanding of mathematical principles (Hajerina et al., 2022).

Operational errors involve incorrect calculations of first and second derivatives, errors in simplifying equations, and mistakes in factoring roots. For instance, one student wrote the second derivative as $f''(x) = 3x - 6$, whereas the correct answer was $6x - 6$. This error demonstrates a lack of carefulness and mastery of basic calculus procedures. These findings align with Manurung et al., who showed that procedural and operational errors frequently occur among mathematics education students (Manurung et al., 2024).

Factual errors relate to the use of mathematical symbols, such as misplacing "<" or ">", which directly affects the interpretation of function intervals. These errors highlight the importance of careful reading of instructions and correct application of notation.

Theoretically, these findings affirm that error analysis can serve as a diagnostic tool to understand students' learning obstacles in calculus and provide a basis for developing more effective teaching strategies. Practically, the results suggest that instructors provide repeated exercises on increasing, decreasing, and concavity functions, along with specific feedback on procedural errors, and emphasize conceptual understanding before students tackle operational problems.

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This is also consistent with Yudhi Anggoro, which demonstrated that combining conceptual and procedural practice can significantly reduce student errors (Yudhi Anggoro, 2023a).

Previous studies have shown that students' understanding of algebraic function derivatives still has gaps, particularly in applying the concepts of increasing and decreasing function intervals, which aligns with the findings of this study (Mulyani & Siregar, 2025). Furthermore, the results of this study are consistent with Erliananda et al., who emphasized the importance of problem-solving strategies in derivative topics to identify and correct operational and conceptual errors (Erliananda et al., 2025). Moreover, error analysis based on Bloom's taxonomy (Sakdiah & Siregar, 2025) helps map the types of student errors, supporting the classification of conceptual, operational, and factual errors in this study. In addition, students' critical thinking ability in solving derivative problems, as discussed by Astuti et al., is relevant for understanding the factors causing conceptual errors in increasing, decreasing, and concavity functions (Astuti et al., 2025). This is also related to the study by Harahap et al., which highlighted obstacles in understanding calculus concepts among both students and teachers, providing additional support for the findings on students' errors in determining function intervals and concavity (Harahap et al., 2025).

The findings are consistent with Hajerina et al. who reported that conceptual errors frequently occur when students fail to connect derivative procedures with their mathematical meaning (Hajerina et al., 2022). Similarly, Anggoro found that students often confuse derivative concepts when solving higher-order calculus problems (Yudhi Anggoro, 2023a). The Creative Problem Solving learning model is a method for solving problems creatively (Oktaviana et al., 2017). However, unlike previous studies that focused on differentiation in general, the present study specifically identifies how conceptual misunderstandings emerge in determining increasing functions, decreasing functions, and function concavity.

This study contributes to the literature on calculus error analysis by providing a detailed examination of conceptual, operational, and factual errors in three derivative application topics: increasing functions, decreasing functions, and function concavity. Unlike previous studies that predominantly examined general differentiation skills, this study highlights the specific difficulties students encounter when interpreting derivative information to analyze function behavior.

From a pedagogical perspective, the findings suggest that calculus instruction should place greater emphasis on conceptual interpretation alongside procedural practice. Lecturers are encouraged to integrate error-based learning activities, reflective discussions, and diagnostic assessments to help students recognize and correct misconceptions before they become deeply rooted.

Conclusion and Suggestion

This study aimed to identify the types of errors made by mathematics education students in solving calculus problems involving increasing functions, decreasing functions, and function concavity, as well as to examine the factors contributing to these errors. The findings revealed that students' errors can be classified into three categories: conceptual errors, operational errors, and factual errors. Among these categories, operational errors were the most dominant,

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indicating that many students experienced difficulties in accurately applying calculus procedures despite having partial conceptual understanding. The results also showed that students' errors were influenced by insufficient conceptual understanding, lack of carefulness, and ineffective problem-solving practices.

These findings highlight the importance of strengthening both conceptual understanding and procedural fluency in calculus learning, particularly in topics related to derivative applications. Error analysis proved useful for identifying students' learning difficulties and providing information that can support instructional improvement.

Based on the findings, lecturers are encouraged to incorporate diagnostic assessments, error-analysis activities, and reflective learning strategies into calculus instruction to help students recognize and correct their mistakes. Future research may investigate the effectiveness of specific instructional interventions designed to reduce conceptual and operational errors, involve larger and more diverse participant groups, or examine students' errors in other advanced calculus topics to obtain a broader understanding of learning difficulties in higher mathematics.

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