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MATHEMATICAL PROCEDURAL FLUENCY OF STUDENTS IN SOLVING MATRIX PROBLEMS

I. Kristin[1], Yulis Jamiah[2], Dian Ahmad[3*]

[1,2,3] Mathematics Education Study Program, FKIP Universitas Tanjungpura, Pontianak, Indonesia
*Corresponding author. Jl. Prof. Dr. H. Hadari Nawawi, Pontianak, Kalimantan, Barat, Indonesia

E-mail: kkristin480@gmail.com [1]
yulis.jamiah@fkip.untan.ac.id [2]
dian.ahmad.budhysatrya@fkip.untan.ac.id [3]

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ABSTRACT
Mathematical procedural fluency is the ability of students to perform procedures flexibly, efficiently, and accurately. This study aims to determine the mathematical procedural fluency of students in solving matrix problems in class XI at SMK Negeri 1 Pontianak. The method used in this research is a descriptive method with a case study approach. There are 25 students from class XI Accounting at SMK Negeri 1 Pontianak as the subjects. Data collection techniques include measurement through descriptive tests, direct communication in the form of interviews, and documentation. The results show that the mathematical procedural fluency of students in solving matrix problems is classified as moderate, with a percentage of 64%. The conclusion of this study indicates that the mathematical procedural fluency is fairly good in solving matrix problems, but still needs improvement through more effective learning strategies focused on developing procedural mathematical abilities.
Keywords: accurate; efficient; flexible; problem solving; procedural fluency

ABSTRAK
Kelancaran procedural matematis merupakan kemampuan siswa dalam melakukan prosdur secara fleksibel, efisien dan akurat. Penelitian ini bertujuan untuk mengetahui kelancaran prosedural matematis siswa dalam menyelesaikan masalah pada materi matriks kelas XI SMK Negeri 1 Pontianak. Metode yang digunakan dalam penelitian adalah metode deskriptif dengan pendekatan studi kasus. Dalam penelitian ini ada 25 siswa dari kelas XI Akuntansi SMK Negeri 1 Pontianak sebagai subjeknya. Teknik pengumpulan data yaitu meliputi pengukuran melalui tes uraian, komunikasi langsung berupa wawancara, dan dokumentasi. Hasil penelitian menunjukkan bahwa kelancaran prosedural matematis siswa dalam pemecahan masalah khususnya pada materi matriks tergolong sedang dengan persentase 64 %. Kesimpulan penelitian ini menunjukkan bahwa kelancaran procedural matematis yang cukup baik dalam pemecahan masalah matriks namun masih perlu peningkatan melalui strategi pembelajaran yang lebih efektif dan terfokus pada pengembangan kemampuan prosedural matematis.
Kata kunci*: akurat; efisien; fleksibel; kelancaran prosedural; pemecahan masalah*

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Introduction

Mathematics plays an important role in everyday life as well as in the world of education. The goal of mathematics learning is to develop problem-solving skills related to everyday life (Jamiah & Pasaribu, 2023). This is in line with the goal of mathematics stated by Sagita et al. (2023) which states that one of the goals of mathematics learning is for students to develop the ability to solve problems, which includes understanding the problem, designing a mathematical model, solving the model, and interpreting the solution obtained.

Problem-solving ability is the ability possessed by a person to solve problems when the right solution has not yet been found and can be applied directly (Suryani et al., 2020). To find the appropriate solution in overcoming a problem, a process is certainly needed. This problem-solving process involves a certain approach or strategy. In mathematics learning, the use of appropriate methods, procedures, and strategies is highly emphasized to support the problem-solving process (Rahmatiya & Miatun, 2020; Fadzil & Osman, 2025).

The key to success in the problem-solving process that must be considered is procedure. The selection of the right procedure and its correct application is one aspect of mathematical skills. Kilpatrick et al. (2001) say that mathematical proficiency has five standards, one of which students must master is procedural fluency. One area that utilizes mathematics to solve problems is the field of vocational expertise. Vocational high school (in Indonesia SMK) is a level of secondary school that prepares its students to work according to their expertise.

According to Inayah et al. (2020) & Carolin et al. (2025), procedural fluency is very much needed by SMK students, because procedural fluency is not only useful in mathematics subjects but also useful in other subjects, especially in productive subjects. Productive subjects emphasize practicum more than theory, of course in conducting practicum there are procedures that must be carried out. The importance of procedural fluency is because it can help students in carrying out their tasks efficiently and accurately.

Kilpatrick et al. (2001) state that procedural fluency is the ability to execute procedures accurately, flexibly, accurately, and efficiently. Without sufficient procedural fluency, students will have difficulty deepening their understanding of mathematical concepts or solving mathematical problems. Students who understand the material can adjust or change procedures to make them easier to apply, while students who only memorize procedures tend to only be able to apply them without deep understanding.

Research conducted by Anggita et al. (2018) discusses procedural fluency in the material of root forms. From the results of their research, they found that only 40% of students can understand conceptually and 31% of students can perform procedures fluently in the material of root forms in class X SMA. Another research by Damayanti et al. (2018) on procedural fluency in solving algebraic operation problems. They found that the procedural fluency of students is still not fluent, as evident from their errors in solving algebraic operation problems. In addition, this is also reflected in the attitude of students who are hesitant and often ask whether the answers they write are correct or not. Then, in research by Sari et al. (2018), which also discusses procedural fluency in solving quadratic equation problems, it is stated that most students are not yet able to solve quadratic equation problems

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in at least two ways, are not yet able to streamline steps, are not yet able to perform correct calculations, and do not write the final conclusion of the problem being asked.

In addition, research by Haryandika et al. (2017) discusses procedural fluency on exponential equation material. The results of their research show a tendency that students lack mathematical procedural fluency in learning mathematics. This is observed from the behavior of students who often ask what steps to take to solve the problems being worked on. Research by Pratidiana & Muhayatun (2021) & Martin (2025) discusses students' procedural fluency in solving linear programming problems. The results of their research show that only one person meets all indicators of procedural fluency.

Based on the researcher's experience during the teaching practice at SMKN 1 Pontianak, it was seen from the results of students' work in solving problems that many students are still wrong in carrying out procedures, lack accuracy in calculations, so the results obtained are not as expected. Based on the problems found from previous research, that procedural fluency is still low. Therefore, researchers need to conduct further research on procedural fluency. In this research, researchers used matrix material to see students' procedural fluency in solving problems. This is done because in solving matrix problems, systematic and structured procedures are very important to avoid errors, increase efficiency, and produce accurate answers. In addition, from the results of research, matrix material has not been widely used in research on procedural fluency.

This research aims to determine the mathematical procedural fluency of students in problem-solving on matrix material.

Research Methods

Type and Research Approach

The type of research used is qualitative research with a descriptive method. The form of research is a case study. Case study research is an intensive examination of a specific object, studying it as a case (Suharsimi, 2010). With the aim of explaining the mathematical procedural fluency of students in solving matrix material problems.

Subjects and Objects of Research

The subjects in this research are students of class XI majoring in Accounting (AKT) 4 at SMK, totaling 25 students. The interview respondents consisted of 6 students, selected using purposive sampling technique, which is a sampling technique adjusted to specific objectives and considerations (Sugiyono, 2013). The object of this research is the mathematical procedural fluency of students in solving problems.

Techniques and Data Collection Tools

The data collection techniques in this research include measurement techniques with essay questions, interviews, and documentation. The data collection tools used are the Student Procedural Fluency Test Sheet and interview guidelines.

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Data Analysis Techniques

Data analysis was carried out through three stages, namely data reduction, data display, and conclusion drawing/verification. At the reduction stage, the data obtained through test results and interviews were then analyzed by grouping research subjects into categories according to procedural fluency indicators. Data display is in the form of narratives, images, and graphs.

Data display is done to facilitate understanding of what is happening, and to plan the next step based on what has been understood (Miles & Huberman, 2007). The data display in this study is in the form of narratives, images, and graphs. The data display in narrative form is the process of students solving procedural fluency problems in problem-solving from interview data. The data display in graph form is the result of procedural fluency test data. The data display in image form is the result of students' answers on the procedural fluency test. The conclusion drawing in this study is how the procedural fluency of students in problem-solving on matrix material in class XI SMK Negeri.

Research Procedure

The following are the research procedures presented in Table 1.

Table 1. Research procedure

Preparation	Implementation	Final
Designing research design	Giving tests to class XI AKT 4 students	Analyzing procedural fluency test results and student interview results
Research design seminar	Conducting interviews	Drawing conclusions
Instrument validation		Compiling reports
Research permit arrangements		
Trial of questions		

Results and Discussion

Research Results

This research was conducted at SMKN 1 Pontianak on class XI Accounting students, attended by 25 students. Data on students' mathematical procedural fluency test results were obtained using an essay test consisting of 2 questions. The scoring of the procedural fluency test was based on an assessment rubric. The overall results of the procedural fluency test analysis fall into the medium category with a percentage of 64%. The test score results for each indicator can be seen in Figure 1 below. Based on the figure, students' mathematical procedural fluency for each indicator falls into the medium category. This means students have fairly good abilities in achieving flexibility, efficiency, and accuracy, but there is still room for improvement and enhancement. The following are student scores for each indicator of procedural fluency shown in Figure 1.

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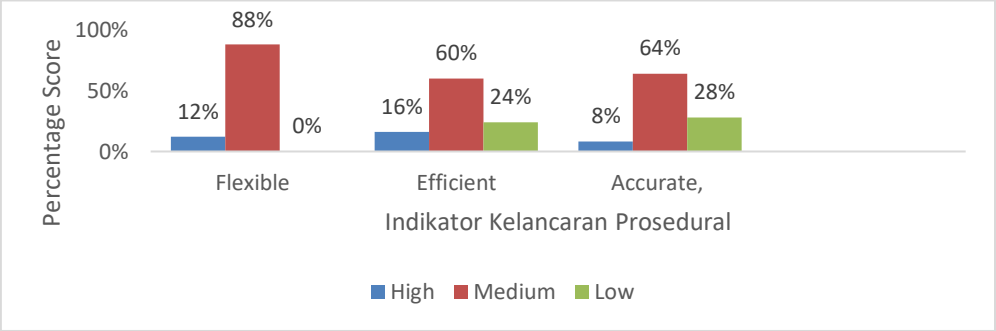

Figure 1. Achievement of student scores on each procedural fluency indicator

Based on Figure 1., the procedural fluency test results, six students were selected for interviews, namely G, BY, V, FM, A, and SR. Here are the results of the analysis of students' procedural fluency.

1. Based on the test results, G was able to solve both procedural fluency test questions flexibly, efficiently, and accurately. G used two methods to solve each problem, completing the questions as briefly as possible with accurate calculations. Based on the procedural fluency indicators used in this study, G met all three indicators of procedural fluency.
2. BY can solve problems flexibly, efficiently, and accurately. Meets procedural fluency indicators.
3. V has not been able to solve problems flexibly and accurately. Has not met the procedural fluency indicators.
4. FM only worked on one problem, using one method, and has not streamlined the steps. Has not met the procedural fluency indicators.
5. A has not met the procedural fluency indicators. It can be seen that A only worked on the problem using one method, namely the determinant method, but A was not able to complete it to get the correct answer. A streamlined the steps in that method but was still inaccurate in the calculations.
6. SR was not able to solve the problem using the two requested methods. Where SR tried to work on using the determinant method, but did not get the correct final answer. This shows that SR has not met the three indicators of procedural fluency.

Discussion

The data from the students' mathematical procedural fluency test results are divided into 3 criteria: high, medium, and low for each procedural fluency indicator, namely flexible, efficient, and accurate. The categorization of these criteria is based on the score results obtained by the students. The interviews conducted aim to further explore the results of the students' mathematical procedural fluency tests. Here is the discussion of the results of the mathematical procedural fluency test in solving matrix material problems and the interview results for each indicator.

Flexible

The flexible aspect in this study is seen from how students solve problems using the two requested methods, namely the determinant method and matrix

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inverse. Students are said to be flexible if they are able to solve problems using both methods correctly. There are some students who are already able to solve problems using more than one method correctly, although there are also those who work on using two methods but only one is correct.

Based on the procedural fluency test results of the six selected research subjects, there are two students who were able to solve problems using both requested methods, namely G and BY. G and BY were able to solve problems using the determinant method and inverse, indicating that both have cognitive flexibility. Cognitive flexibility occurs when students are able to change ideas and solve problems with various approaches (Rahayuningsih et al., 2020; Honra & Monterola, 2025; Orakcı & Khalili, 2025; Uras, 2025). The higher the cognitive flexibility that students have, the higher their problem-solving abilities (Oktaviani et al., 2021). As for the FM subject, he was only able to solve the problem using one method, namely the determinant method. While the other three subjects, namely V, A, and SR, tried to solve the problem using the determinant method, but were not able to complete it to get the correct final answer. From this, it can be seen that students' cognitive flexibility is still lacking. According to Oktaviani et al. (2021), low cognitive flexibility will lead to failure in realizing that the problem at hand is a new problem, and difficulty in adjusting to the situation.

Based on the interview results, students worked on using the method they thought was easier or they could do, namely the determinant method. The results of interviews conducted on FM, V, A, and SR for the inverse matrix method, they knew the procedure but did not understand how to solve it using that procedure. This indicates a lack of conceptual understanding of the method used. Students who have a lack of conceptual understanding will face difficulties in deepening their understanding of mathematics or solving mathematical problems. As a result, students who only study procedures without conceptual understanding will have difficulty when faced with new mathematical problems (Anggita et al., 2018; Ncube & Luneta, 2025). This is in line with the research results of Kusuma & Masduki (2016) which state that students with strong conceptual understanding will be able to solve a problem using various strategies and procedures.

Efficient

The aspect of efficiency in this study is seen from how students solve problems with the shortest steps but still produce the correct solution. Students are considered efficient if they can solve problems using the shortest possible steps and obtain accurate results.

Based on the procedural fluency test results of the six research subjects, there are two subjects who meet the efficient aspect, namely G and BY. This shows that both have the ability to identify key steps in the procedure, understand the underlying procedures and concepts, and thus can apply them correctly. Students who have cognitive flexibility can choose the right strategy to avoid using unnecessary procedures, making problem-solving more efficient. V and A tried to streamline the procedure but were still inaccurate in calculations. This proves that these students understand efficient steps, but are less skilled in performing calculations. SR and FM have not been able to streamline the procedure. Based on this discussion, it can be concluded that students who have good conceptual

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understanding and flexibility are able to solve problems with short and correct steps.

Accurate

The accurate aspect in this study is seen from the results of students' calculations and the use of appropriate symbols. Students are said to be accurate if they are able to solve problems with accurate calculations and use of symbols. Students often make mistakes when solving problems. According to Siregar (2019), some factors that cause mistakes that students often make when working on math problems include: lack of understanding of symbols, limited understanding of basic concepts, weak ability of students to apply the correct solution process, lack of thoroughness, and sloppy writing, making it difficult for students to read their own handwriting.

Based on the test results, two of the six selected research subjects meet the accurate aspect, namely G and BY, which shows that both have good accuracy in calculations and use of symbols. FM performed calculations correctly, but was less thorough in writing the matrix coefficient symbols, which should be written vertically but were written horizontally as 'Dx, Dy, Dz'. A and V were less thorough in performing calculations, but symbol writing was accurate. SR was less thorough in writing down information from the problem, where "8,500" became "8,500,000", making the given answer incorrect. This is in line with the opinion that has been put forward, that thoroughness affects accuracy.

Conclusion and Suggestions

Based on the results of data analysis, interviews, and discussions that have been conducted, it can be concluded that students' flexibility, efficiency, and accuracy in problem-solving on matrix material fall into the medium category. Therefore, efforts need to be made to improve students' mathematical procedural fluency. Here are some suggestions that researchers can provide based on the findings in this study: 1) Teachers can improve learning by focusing on developing students' flexibility, efficiency, and accuracy in problem-solving. 2) Students need to be given more and varied practice to improve their problem-solving abilities. 3) Developing more effective and interactive learning methods can help improve students' mathematical procedural fluency.

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