

Goodness Of Fit Test In Structural Equation Modeling with Unweighted Least Square (ULS) Estimation Method

Ani Amanathi^{1*}, Eri Setiawan², Mustofa Usman³

^{1*,2,3}Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Lampung, Indonesia

*corresponding author: aniamanathi00@gmail.com

Received June 22, 2023; Received in revised form July 15, 2023; Accepted July 18, 2023 (9pt)

Abstrak. Model persamaan struktural (MPS) adalah metode analisis multivariat yang digunakan untuk menggambarkan hubungan linear secara simultan antara variabel indikator dan variabel laten. Terdapat beberapa metode estimasi pada MPS salah satunya yaitu Unweighted Least Square (ULS). Pada metode tersebut tidak memiliki asumsi khusus dari sebaran peubah. Penelitian ini bertujuan untuk mengestimasi model dengan menggunakan metode ULS serta melihat pengaruh variabel kompetensi pegawai dan fasilitas perpustakaan terhadap kualitas pelayanan di perpustakaan Universitas Lampung. Data yang digunakan adalah data hasil survei kuisioner tentang kualitas pelayanan perpustakaan di Universitas Lampung. Berdasarkan hasil penelitian diperoleh bahwa dari ketiga uji kecocokan yaitu uji kecocokan keseluruhan model, uji kecocokan model struktural dan uji kecocokan model pengukuran dengan menggunakan estimasi ULS sudah menunjukkan hasil yang baik untuk menjelaskan kecocokan antara model dengan data.

Kata kunci: Uji Kecocokan Model; ULS; Model Persamaan Struktural

Abstract. Structural equation model (SEM) is a multivariate analysis method that is used to describe a linear relationship simultaneously between indicator variables and latent variables. There are several estimation methods in SEM, one of them is Unweighted Least Square (ULS). The method doesn't have specific assumptions about the distribution of variables. This study aims to estimate the model using the ULS method and see the influence of employee competency variables and library facilities on the quality of service at the Universitas Lampung library. Survey of quality of service in the library of Lampung University is used in the research. Based on the results of the study, it is found that from the three suitability tests, namely the overall model test, the structural model test and the measurement model test using ULS estimation give good results in explaining the compatibility between the model and observation results.

Keywords: Goodness of Fit Test; ULS; Structural Equation Model



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INTRODUCTION

SEM is a method that was formed because of the problem of measuring a variable where there is a variable that cannot be measured directly (Pagadala et al., 2023). Variables that cannot be measured are called latent variables which require a manifest variable as an indicator or measuring tool for the latent variable. There are several estimation methods in

the SEM method including Maximum Likelihood (ML), Weighted Least Squares (WLS), Generalized Least Squares (GLS), Unweighted Least Squares (ULS) (Ghanbar & Rezvani, 2023). Each method has specificity in its use according to the distribution, sample size and scale of the data, as well as the underlying assumptions. According to Mokhtar et al. (2023) and Tarko (2023) some parameter estimation methods require strict assumptions related to the shape of the distribution and sample size. The ML and GLS methods require the assumption of normality in the data (Du & Bentler, 2022). While the WLS and ULS methods do not require the assumption of normality (Kyriazos & Poga-Kyriazou, 2023; Zulkifli et al., 2023).

ULS is a method that does not require special assumptions from the distribution of observed variables as long as the parameters are identified (Foldnes & Grønneberg, 2022; Hikmah et al., 2023). The ULS method has consistent and unbiased estimator properties and the process is relatively fast due to the simplicity of the method, but the ULS estimator is not an efficient estimator for large data (Chen et al., 2023; Dhaene & Rosseel, 2023). Therefore, this study will discuss the use of theoretical concepts or constructs that cannot be observed directly. In previous research by Mahmoodi et al. (2022), the Structural Equation Modeling method was tested for the analysis of factors affecting the quality of library services with the LISREL 8.80 program. In this study, the fit index test will be carried out with the ULS method using the survey data from the University of Lampung Library Service Quality questionnaire.

METHOD

This study uses primary data with the research population, namely visitors to the University of Lampung library class of 2015 and 2016. From this population, a sample of 100 library visitor respondents will be taken with a purposive sampling technique. The technique is done by taking samples based on certain percentages and considerations. Data collection using a Likert scale questionnaire with 5 categories used, namely strongly disagree (STS), disagree (TS), moderately agree (CS), agree (S) and strongly agree (SS) ranging from 1 to 5. The variables used are 3 latent variables and 12 indicator variables.

The steps in this research are (Robitzsch, 2022):

1. Conducting a survey of visitors to the Universitas Lampung library
2. Inputting questionnaire data with a sample size of 100
3. Testing the validity and reliability of the questionnaire data using SPSS 23
4. Designing Model Specifications: Structural Model and Measurement Model
5. Path diagram construction
Constructing a path diagram is to build a relationship between latent variables, namely ξ_1, ξ_2, η_1
6. Model Estimation
Parameter estimation using the Unweighted Least Square method by minimize half the sum of the squares of the elements of the leftover matrix. The estimates were obtained with the help of the Lisrel 8.80 program.
7. Evaluating the fit test
In general, the test of the overall fit of the model can be seen from the results of Goodness Of Fit (GOF) results, such as Chi Square, RMSEA, AGFI and PGFI (Dash & Paul, 2021; Hidayat & Wulandari, 2022).

RESULT AND DISCUSSION

1. Measurement Model Specification

The measurement model is a model that describes the relationship between latent variables and observed variables. In this study, 12 observed variables were used which consists of 7 observed variables X and 5 observed variables Y is the path diagram:

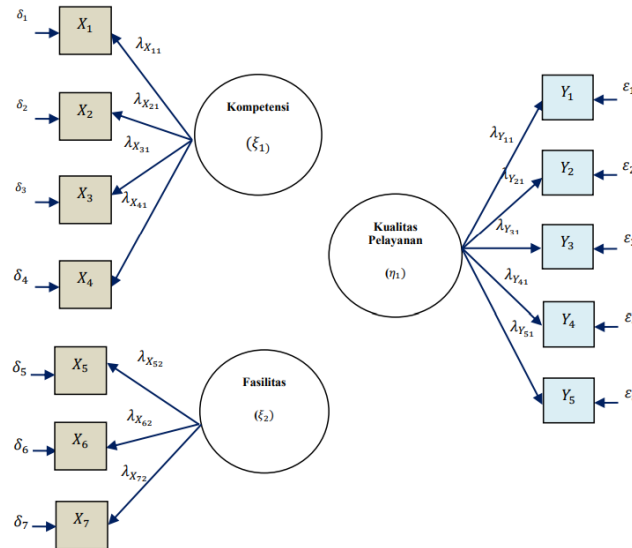


Figure 1. Measurement Model Path Diagram

The model in matrix form becomes:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix} = \begin{bmatrix} \lambda_{X11} & 0 \\ \lambda_{X21} & 0 \\ \lambda_{X31} & 0 \\ \lambda_{X41} & 0 \\ 0 & \lambda_{X52} \\ 0 & \lambda_{X62} \\ 0 & \lambda_{X72} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} = \begin{bmatrix} \lambda_{Y11} \\ \lambda_{Y21} \\ \lambda_{Y31} \\ \lambda_{Y41} \\ \lambda_{Y51} \end{bmatrix} [\eta_1] + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{bmatrix}$$

2. Structural Model Specifications

The structural model is a model that describes the relationship between latent variables. In this study, 3 latent variables were used, consisting of 2 exogenous latent variables and 1 endogenous latent variable. The following is a diagram the path.

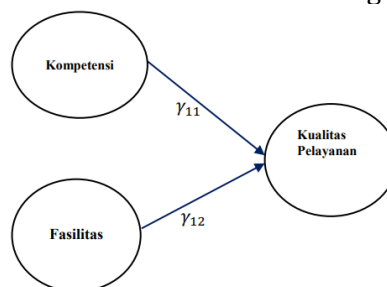


Figure 2. Structural Model Path Diagram

The model in matrix form becomes:

$$[\eta] = [\gamma_{11} \quad \gamma_{12}] \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + [\zeta_1]$$

3. Estimasi Parameter dengan Metode Unweighted Least Square

a. Estimasi Parameter $\hat{\gamma}$, $\hat{\lambda}_x$, and $\hat{\lambda}_y$

(1) Initially, equation of the structural equation model, i.e. :

$$\eta = \gamma_{11} \xi_1 + \gamma_{12} \xi_2 + \zeta_1 \quad (1)$$

(2) With Unweighted Least Squares (ULS) has a fit function inequation as follows:

$$F_{uls} = \frac{1}{2} \text{tr} (\mathbf{S} - \Sigma(\boldsymbol{\theta}))^2 \quad (2)$$

(3) From equations (1) and (2) suppose that the sum of squares of residuals as follows:

$$\begin{aligned} g'f &= \frac{1}{2} \text{tr} \zeta' \zeta \\ &= \frac{1}{2} \text{tr} ((\eta - \hat{\gamma}_{11} \xi_1 + \hat{\gamma}_{12} \xi_2)' (\eta - \hat{\gamma}_{11} \xi_1 + \hat{\gamma}_{12} \xi_2)) \\ &= \frac{1}{2} \text{tr} ((\eta' - \hat{\gamma}_{11}' \xi_1' + \hat{\gamma}_{12}' \xi_2') (\eta - \hat{\gamma}_{11} \xi_1 + \hat{\gamma}_{12} \xi_2)) \\ &= \frac{1}{2} \text{tr} (\eta' \eta - \eta' \hat{\gamma}_{11} \xi_1 + \eta' \hat{\gamma}_{12} \xi_2 - \eta \hat{\gamma}_{11}' \xi_1' + \hat{\gamma}_{11}' \xi_1' \hat{\gamma}_{11} \xi_1 - \\ &\quad \hat{\gamma}_{11}' \xi_1' \hat{\gamma}_{12} \xi_2 + \eta \hat{\gamma}_{12}' \xi_2' - \hat{\gamma}_{11} \xi_1 \hat{\gamma}_{12}' \xi_2' + \hat{\gamma}_{12}' \xi_2' \hat{\gamma}_{12} \xi_2) \\ &= \frac{1}{2} \text{tr} (\eta' \eta - 2 \eta \hat{\gamma}_{11}' \xi_1' + 2 \eta \hat{\gamma}_{12}' \xi_2' + \hat{\gamma}_{11}' \xi_1' \hat{\gamma}_{11} \xi_1 - 2 \hat{\gamma}_{11} \xi_1 \hat{\gamma}_{12}' \xi_2' \\ &\quad + \hat{\gamma}_{12}' \xi_2' \hat{\gamma}_{12} \xi_2) \end{aligned}$$

Since $\eta' \hat{\gamma}_{11} \xi_1 ; \eta' \hat{\gamma}_{12} \xi_2 ; \hat{\gamma}_{11}' \xi_1' \hat{\gamma}_{12} \xi_2$ is a scalar, that form is the same as with its transpose.

(4) To obtain an estimate so that the residual sum of squares is as small as possible, we differentiate $g'f$ against $\hat{\gamma}_{11}$ and against $\hat{\gamma}_{12}$ then we get equation as follows:

$$\frac{\partial g'f}{\partial \hat{\gamma}_{11}} = \text{tr} (-\eta \xi_1' + \hat{\gamma}_{11} \xi_1' \xi_1 - \xi_1 \hat{\gamma}_{12}' \xi_2')$$

and

$$\frac{\partial g'f}{\partial \hat{\gamma}_{12}} = \text{tr} (\eta \xi_2' - \hat{\gamma}_{11} \xi_1 \xi_2' + \xi_2' \hat{\gamma}_{12} \xi_2)$$

by taking $\frac{\partial g'f}{\partial \hat{\gamma}_{11}} = 0$ $\frac{\partial g'f}{\partial \hat{\gamma}_{12}} = 0$ then the equation can be obtained as follows:

$$\text{tr}(-\eta \xi_1' + \hat{\gamma}_{11} \xi_1' \xi_1 - \xi_1 \hat{\gamma}_{12}' \xi_2') = 0$$

$$\text{tr} \hat{\gamma}_{11} \xi_1' \xi_1 = \text{tr}(\eta \xi_1' + \xi_1 \hat{\gamma}_{12}' \xi_2') = 0$$

$$\text{tr}(\xi_1' \xi_1)^{-1} \hat{\gamma}_{11} \text{tr}(\xi_1' \xi_1) = \text{tr}(\eta \xi_1' + \xi_1 \hat{\gamma}_{12}' \xi_2') \text{tr}(\xi_1' \xi_1)^{-1}$$

$$\hat{\gamma}_{11} = \text{tr}(\eta \xi_1' + \xi_1 \hat{\gamma}_{12}' \xi_2') \text{tr}(\xi_1' \xi_1)^{-1}$$

and

$$\text{tr}(\eta \xi_2' - \hat{\gamma}_{11} \xi_1 \xi_2' + \xi_2' \hat{\gamma}_{12} \xi_2) = 0$$

$$\text{tr} \xi_2' \hat{\gamma}_{12} \xi_2 = \text{tr}(\hat{\gamma}_{11} \xi_1 \xi_2' - \eta \xi_2')$$

$$\text{tr}(\xi_2' \xi_2)^{-1} \hat{\gamma}_{12} \text{tr}(\xi_2' \xi_2) = \text{tr}(\hat{\gamma}_{11} \xi_1 \xi_2' - \eta \xi_2') \text{tr}(\xi_2' \xi_2)^{-1}$$

$$\hat{\gamma}_{12} = \text{tr}(\hat{\gamma}_{11} \xi_1 \xi_2' - \eta \xi_2') \text{tr}(\xi_2' \xi_2)^{-1}$$

- (5) As for factor loading, it can be estimated using ULS, initially the measurement model equations in equations, namely:

$$X = \lambda_X \xi + \delta \tag{3}$$

$$Y = \lambda_Y \eta + \varepsilon \tag{4}$$

- (6) From equations (2) and (3) suppose that $h'f$, the sum of of squares of residuals as follows:

$$\begin{aligned} h'f &= \frac{1}{2} \text{tr} \delta' \delta \\ &= \frac{1}{2} \text{tr} ((X - \hat{\lambda}_X \xi)' (X - \hat{\lambda}_X \xi)) \\ &= \frac{1}{2} \text{tr} ((X' - \hat{\lambda}_X' \xi') (X - \hat{\lambda}_X \xi)) \\ &= \frac{1}{2} \text{tr} (X' X - X' \hat{\lambda}_X \xi - X \hat{\lambda}_X' \xi' + \lambda_X' \xi' \hat{\lambda}_X \xi) \\ &= \frac{1}{2} \text{tr} (X' X - 2 X \hat{\lambda}_X' \xi' + \hat{\lambda}_X' \xi' \hat{\lambda}_X \xi) \\ &= \text{tr} (\frac{1}{2} X' X - X \hat{\lambda}_X' \xi' + \frac{1}{2} \hat{\lambda}_X' \xi' \hat{\lambda}_X \xi) \end{aligned}$$

From equations (2) and (4) suppose that $k'f$, the sum of of squares of residuals as follows:

$$\begin{aligned}
 k'f &= \text{tr} \frac{1}{2} \varepsilon' \varepsilon \\
 &= \frac{1}{2} \text{tr} ((Y - \hat{\lambda}_Y \eta)' (Y - \hat{\lambda}_Y \eta)) \\
 &= \frac{1}{2} \text{tr} ((Y' - \hat{\lambda}_Y' \eta') (Y - \hat{\lambda}_Y \eta)) \\
 &= \frac{1}{2} \text{tr} (Y' Y - Y' \hat{\lambda}_Y \eta - Y \hat{\lambda}_Y' \eta' + \hat{\lambda}_Y' \eta' \hat{\lambda}_Y \eta) \\
 &= \frac{1}{2} \text{tr} (Y' Y - 2Y \hat{\lambda}_Y' \eta' + \hat{\lambda}_Y' \eta' \hat{\lambda}_Y \eta) \\
 &= \text{tr} (\frac{1}{2} Y' Y - Y \hat{\lambda}_Y' \eta' + \frac{1}{2} \hat{\lambda}_Y' \eta' \hat{\lambda}_Y \eta)
 \end{aligned}$$

Since $X' \hat{\lambda}_X \xi$; $Y' \hat{\lambda}_Y \eta$ is a scalar, that form is equal to its transpose.

- (7) To obtain the value of the estimator so that the sum of the residual squares is as small as possible as possible, then we differentiate $h'f$ against $\hat{\lambda}_X$ and $k'f$ against $\hat{\lambda}_Y$, then we get the following equation:

$$\frac{\partial h'f}{\partial \hat{\lambda}_X} = \text{tr} (-X\xi' + \hat{\lambda}_X \xi \xi')$$

and

$$\frac{\partial k'f}{\partial \hat{\lambda}_Y} = \text{tr} (-Y\eta' + \hat{\lambda}_Y \eta \eta')$$

by taking $\frac{\partial h'f}{\partial \hat{\lambda}_X} = 0$ and $\frac{\partial k'f}{\partial \hat{\lambda}_Y} = 0$ then an equation can be obtained follows:

$$\text{tr} (-X\xi' + \hat{\lambda}_X \xi \xi') = 0$$

$$\text{tr}(\hat{\lambda}_X \xi \xi') = \text{tr}(X\xi')$$

$$\text{tr}(\xi \xi')^{-1} \hat{\lambda}_X \text{tr}(\xi \xi') = \text{tr} X \xi' \text{tr}(\xi \xi')^{-1}$$

$$\hat{\lambda}_X = \text{tr} X \xi' \text{tr}(\xi \xi')^{-1}$$

and

$$\text{tr}(-Y\eta' + \hat{\lambda}_Y \eta \eta') = 0$$

$$\text{tr}(\hat{\lambda}_Y \eta \eta') = \text{tr}(Y\eta')$$

$$\text{tr}(\eta \eta')^{-1} \hat{\lambda}_Y \text{tr}(\eta \eta') = \text{tr} Y \eta' \text{tr}(\eta \eta')^{-1}$$

$$\hat{\lambda}_Y = \text{tr} Y \eta' \text{tr}(\eta \eta')^{-1}$$

Based on the parameter estimation results above, the parameter estimation in the structural equation model cannot be done by manual calculation because the structural equation model is analyzed simultaneously so that the population covariance ($\Sigma(\theta)$) is close to or equal to the sample covariance (S). In addition, the structural equation model also involves many equations and involves many parameters so it will be difficult to obtain the parameter estimation value. Therefore, the structural equation model analysis was carried out with the help of LISREL software.

4. Estimation of Parameter Values in the Model Using Lisrel Software 8.80

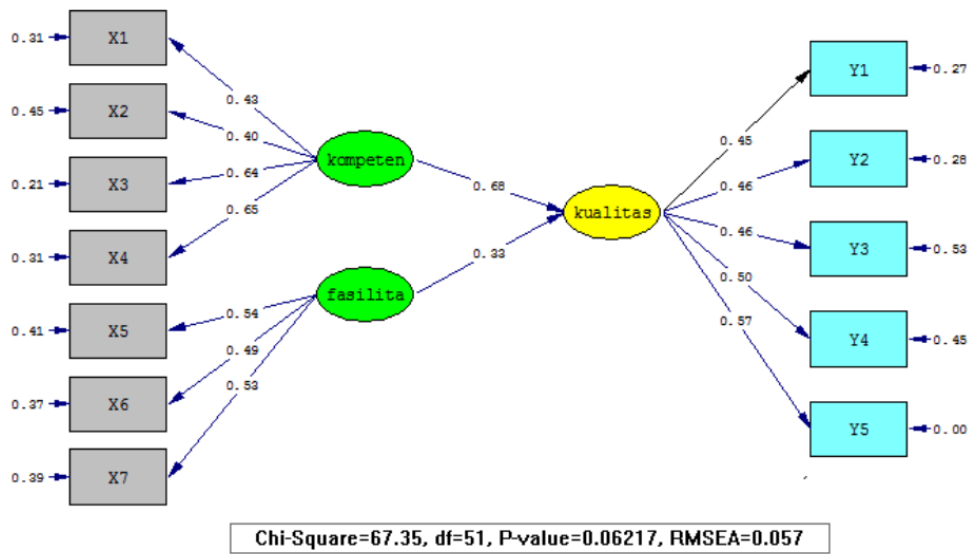


Figure 3. Path Diagram of Estimation Results

Based on Figure 3, the estimated value for all parameters at size 100, which γ_{11} is 0.68. This means that the value of the relationship between employee competence and service quality is 0.68, while for γ_{21} is 0.33, which means that the value of the relationship between library facilities and service quality is 0.33.

From the parameter estimation results obtained, the equation can be arranged for the structural model and measurement model.

- Structural Model

$$\eta = 0,68\xi_1 + 0,33\xi_2 + \zeta_1$$

- Measurement Model

$$X_1 = 0,43\xi_1 + 0,31$$

$$Y_1 = 0,45\eta_1 + 0,27$$

$$X_2 = 0,40\xi_1 + 0,45$$

$$Y_2 = 0,46\eta_1 + 0,28$$

$$X_3 = 0,64\xi_1 + 0,21$$

$$Y_3 = 0,46\eta_1 + 0,53$$

$$X_4 = 0,65\xi_1 + 0,31$$

$$Y_4 = 0,50\eta_1 + 0,45$$

$$X_5 = 0,54\xi_2 + 0,41$$

$$Y_5 = 0,57\eta_1 + 0,32$$

$$X_6 = 0,49\xi_2 + 0,37$$

$$X_7 = 0,53\xi_2 + 0,39$$

5. Overall Model Fit Test

The overall model fit test is used to evaluate the general degree of fit by looking at the goodness of fit results. Overall There are several criteria for testing a model, namely the Absolute Fit Test, Incremental Fit Test and Parsimony Fit Test. Absolute Fit Test, Incremental Fit Test and Parsimony Fit Test. In the absolute fit test The absolute fit test

consists of Chi Square and RMSEA. While in the incremental fit test incremental fit test is the NFI and in the parsimony fit test is the PNFI. The following are the results of testing the overall suitability of the model can be seen in Table 1.

Table 1. Overall Model Fit Test

GOF	Target Match Rate	Estimation Results	Rate Match
Chi-Square P-value	Small value $P > 0,05$	67,35 (P = 0,062)	Good
RMSEA	$0,05 \leq RMSEA \leq 0,08$ good fit	0,057	Good
NFI	$NFI \geq 0,90$ good fit	0,99	Good
PNFI	The bigger the better	0,77	Good

Based on the results of the overall model fit test in Table 1, it can be seen that that the chi-square value above is 67,35 and has a p-value of 0,062 ($p\text{-value} > 0,05$) which means the fit is good. For the RMSEA value value shows a good fit as well with a result of 0,057. Likewise, the NFI and PNFI results show a good fit model with each result obtained of 0,99 and 0,05. with each result obtained of 0,99 and 0,77. So that it can be concluded that the overall fit of the model is good.

CONCLUSION

The results of the three fit tests, namely the overall model fit test, the structural model fit test and the measurement model fit test with the Unweighted Least Square estimation method at a sample size of 100 have shown good results to explain the fit between the model and the data. Service quality variables at Lampung University Library (η_1) can be explained by employee competence variables (ξ_1) and library facilities (ξ_2) by 85%, while the remaining 15% is explained by other factors outside the model.

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