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Maximum Likelihood Estimation Approach using the CB-SEM Method: Case Study of Service Quality

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Abstrak. *Penelitian ini bertujuan untuk menganalisis pengaruh tingkat pelayanan, kepuasan dan loyalitas pengunjung kawasan wisata Pantai Sariringgung. Tipe SEM yang digunakan pada penelitian ini yaitu pendekatancovarian based dengan metode estimasi maximum likelihood. Hasil Penelitian ini menunjukkan bahwa pengaruh langsung tingkat pelayanan ke kepuasan sebesar 77%, tingkat pelayanan terhadap loyalitas sebesar 35%, sedangkan pengaruh tidak langsung tingkat pelayanan terhadap loyalitas melalui kepuasan sebesar 38,5%. Kemudian besar pengaruh total tingkat pelayanan terhadap loyalitas pelanggan melalui kepuasan yaitu sebesar 73,5%.*

Kata kunci*: CB-SEM; ML; Pengaruh Total; SEM*

The purpose of this study to analyze the service level, satisfaction and loyalty of Sariringgung Beach visitors.Covarian based approach to estimatemaximum likelihood method in the service levelto the satisfaction and loyalty of visitorstourism area of Sariringgung Beach is used. The results of this study indicate that the direct effect of service level to satisfaction is 77%, service level to loyalty is 75%. Whereas the indirect effect of service level on loyalty through satisfaction is 38,5%. Then the total effect of service level on customer loyalty through satisfaction is 73,5%.

Keywords: CB-SEM; ML; Total Effect; SEM.

INTRODUCTION

The level of service, satisfaction and loyalty cannot be measured directly. Therefore, to analyze causal relationships in structural unobserved variables, analytical methods are needed that take into account the nature of these relationships (Rasoolimanesh et al., 2021; Zhang et al., 2021). One method that can be used to analyze causal relationships as discussed above is Structural Equation Modeling (SEM) (Bullock et al., 1994; Lowry & Gaskin, 2014). According to Legate et al. (2023) and Zyphur et al. (2023) One of the advantages of SEM is the ability to model constructs as latent variables or variables that are not measured directly, but are estimated in the model from the measured variables which are assumed to have a relationship with the latent variable.

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Generally, there are two types of SEM that are widely known, namely Covariance Based-Structural Equation Modeling (CB-SEM) developed by Jöreskog et al. (2016) and Partial Least Square Structural Equation Modeling (PLS-SEM) often called variance or component-based structural equation modeling developed by Hair Jr et al. (2017). CB-SEM aims to estimate structural models based on strong theoretical studies to test causal relationships between constructs as well as measure the feasibility of the model and confirm it according to empirical data (Daryono et al., 2023; Hidayat & Wulandari, 2022). CB-SEM demands a strong theoretical base, meets various parametric assumptions and meets model feasibility tests (goodness of fit). Seeing this phenomenon, researchers are interested in examining the influence of service levels on visitor satisfaction and loyalty in the Sariringgung Beach tourist area using the Maximum Likelihood estimation method with the CB-SEM approach to test the theory and obtain the truth of the test with a series of complex analyzes.

METHOD

The data needed in the research is primary data with an infinite population, namely visitors to the Sariringgung Beach tourist area in 2018, so a sample size of (n = 200) was taken by applying a simple random sampling technique, namely random sampling of visitors to Sariringgung Beach. Primary data is data collected directly by the researchers themselves by giving questionnaires to respondents who visited Sariringgung Beach as a case study.

The steps in this research method are as follows:

1. Model Specifications
   Designing structural models and measurement models used to carry out testing. This research consists of 3 latent variables, namely loyalty (η_1) and satisfaction (η_2) and service (ξ_1) and 13 variables observed are X1, X2, X3, X4, X5, X6.
2. Construction of a path diagram
   Constructing a path diagram means building relationships between latent variables, namely $\xi 1$, $\eta 1$, $\eta 2$ and writing parameter symbols for each loading factor value.
3. Test the overall suitability of the model
   Evaluate the results of the goodness-of-fit test to see the feasibility of the model using the maximum likelihood estimation method (Deva & Husein, 2017; Levene & Kononovicius, 2021). This comparison was carried out by looking at the Goodness Of Fit (GOF) values in the Chi-Square, NCP, GFI, RMSEA, AGFI, PNFI and NFI test statistics.
4. Parameter Estimation
   This research uses the maximum likelihood method with the following steps:
   a. Forming a likelihood function derived from structural equations.
   b. Maximize the function obtained to obtain estimated parameters.
   c. Find the first derivative of the maximum likelihood function ln for the parameter to be estimated and equate it to zero.
5. Using Lisrel 8.80 software to obtain estimated values for parameters γ, β and λ.
6. Testing the significance of parameters in the measurement model
   Evaluation is carried out by looking at the t value of factor loadings ≥1.96 and standard factor loadings ≥0.05.
7. View direct, indirect effects and calculate the total effect between latent variables.
8. Evaluation of the CB-SEM model with Lisrel 8.80 software based on the coefficient of determination value in the model.

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RESULT AND DISCUSSION

a. Estimated Parameters

Suppose η and ξ are multinormal random variables of size n with η^T = ($\eta_1, \eta_2, \dots, \eta_m$) and ξ^T = ($\xi_1, \xi_2, \dots, \xi_n$), because they are assumed to be normal then η~N (**β**η-**Γ**ξ; **Σ**). So the probability density function is:

$$F(\boldsymbol{\beta}\eta-\boldsymbol{\Gamma}\xi;\boldsymbol{\Sigma}) = \frac{1}{\sqrt{(2\pi)^{p+q}|\boldsymbol{\Sigma}|}}\exp\left\{-\frac{1}{2\boldsymbol{\Sigma}}(\eta-\boldsymbol{\beta}\eta-\boldsymbol{\Gamma}\xi)'(\eta-\boldsymbol{\beta}\eta-\boldsymbol{\Gamma}\xi)\right\}$$

If x= β η-**Γ**ξ then the joint density function for stochastically independent and identical random samples at x is as follows:

$$L= f(x_1),f(x_2),\dots, f(x_n)$$

with the likelihood function:

$$L(\theta)= \prod_{i=1}^{n} f(x_i)$$

$$= \ln\left\{(2\pi)^{\frac{-n(p+q)}{2}}|\boldsymbol{\Sigma}|^{-\frac{n}{2}}\exp\left\{-\frac{1}{2\boldsymbol{\Sigma}}(\eta-\boldsymbol{\beta}\eta-\boldsymbol{\Gamma}\xi)'(\eta-\boldsymbol{\beta}\eta-\boldsymbol{\Gamma}\xi)\right\}\right\}$$

$$= \ln\left((2\pi)^{\frac{-n(p+q)}{2}}\right) + \ln\left(|\boldsymbol{\Sigma}|^{-\frac{n}{2}}\right) + \ln\ \exp\ \left\{-\frac{1}{2\boldsymbol{\Sigma}}(\eta'\eta-2\eta'\boldsymbol{\beta}'\eta-2\xi'\boldsymbol{\Gamma}'\eta+\eta'\boldsymbol{\beta}'\boldsymbol{\beta}\eta+\boldsymbol{2}\eta'\boldsymbol{\beta}'\boldsymbol{\Gamma}\xi+\xi'\boldsymbol{\Gamma}'\boldsymbol{\Gamma}\xi)\right\}$$

$$= \frac{-n(p+q)}{2}\ln(2\pi)-\frac{n}{2}\ln(|\boldsymbol{\Sigma}|)-\frac{1}{2\boldsymbol{\Sigma}}\eta'\eta+\frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}'\eta+\frac{1}{\boldsymbol{\Sigma}}\xi'\boldsymbol{\Gamma}'\eta-\frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}'\boldsymbol{\beta}-\frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}'\boldsymbol{\Gamma}\xi-\frac{1}{2\boldsymbol{\Sigma}}\xi'\boldsymbol{\Gamma}'\boldsymbol{\Gamma}\xi$$

Estimation of parameter β

$$0= \frac{\partial L(\theta)}{\partial \boldsymbol{\beta}}$$

$$= \frac{\partial\left(\frac{-n(p+q)}{2}\ln(2\pi)-\frac{n}{2}\ln(|\boldsymbol{\Sigma}|)-\frac{1}{2\boldsymbol{\Sigma}}\eta'\eta+\frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}'\eta+\frac{1}{\boldsymbol{\Sigma}}\xi'\boldsymbol{\Gamma}'\eta-\frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}'\boldsymbol{\beta}\eta-\frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}'\boldsymbol{\Gamma}\xi-\frac{1}{2\boldsymbol{\Sigma}}\xi'\boldsymbol{\Gamma}'\boldsymbol{\Gamma}\xi\right)}{\partial \boldsymbol{\beta}};\text{with } \frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}'\boldsymbol{\beta}\eta$$

Based on the derivative properties of the matrix **f(x)= x′Ax** then $\frac{\partial f}{\partial \boldsymbol{\beta}}$= **(A+A′)x**, so that the derivative of **β** is obtained as follows:

$$0= -0 - 0 - 0 + \frac{1}{\boldsymbol{\Sigma}}\eta'\eta + 0 - \frac{1}{2\boldsymbol{\Sigma}}(\eta'\boldsymbol{\beta}\eta + (\eta'\boldsymbol{\beta}'\eta)') - \frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\Gamma}\xi - 0$$

$$0= \frac{1}{\boldsymbol{\Sigma}}\eta'\eta - \frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}\eta - \frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}\eta - \frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\Gamma}\xi$$

$$0= \frac{1}{\boldsymbol{\Sigma}}\eta'\eta - \frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\beta}\eta - \frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\Gamma}\xi$$

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$$\widehat{\boldsymbol{\beta}}\left(\frac{1}{\Sigma}\,\eta'\eta\right)=\frac{1}{\Sigma}\eta'\eta - \frac{1}{\Sigma}\,\eta'\widehat{\boldsymbol{\gamma}}\xi$$

$$\widehat{\boldsymbol{\beta}}= (\eta'\eta - \eta'\widehat{\boldsymbol{\gamma}}\xi)(\eta'\eta)^{-1}$$

Estimation of parameter γ

$$0=\frac{\partial L(\theta)}{\partial \boldsymbol{\gamma}}$$

$$=\frac{\partial\left(\frac{-n(p+q)}{2}\ln(2\pi)-\frac{n}{2}\ln(|\boldsymbol{\Sigma}|)-\frac{1}{2\Sigma}\eta'\eta+\frac{1}{\Sigma}\eta'\boldsymbol{\beta}'\eta+\frac{1}{\Sigma}\xi'\boldsymbol{\Gamma}'\eta-\frac{1}{2\Sigma}\eta'\boldsymbol{\beta}'\boldsymbol{\beta}\eta-\frac{1}{\Sigma}\eta'\boldsymbol{\beta}'\boldsymbol{\Gamma}\xi-\frac{1}{2\Sigma}\xi'\boldsymbol{\Gamma}'\boldsymbol{\Gamma}\xi\right)}{\partial\boldsymbol{\gamma}};\text{ with } \frac{1}{2\Sigma}\,\xi'\boldsymbol{\Gamma}'\boldsymbol{\Gamma}\xi$$

Based on the derivative properties of the matrix **f(x)= x′Ax** then $\frac{\partial f}{\partial \beta}$= **(A+A′)x**, so that the derivative of γ is obtained as follows:

$$0= 0- 0- 0+ 0 +\frac{1}{\Sigma}\,\xi'\eta- 0-\frac{1}{\Sigma}\,\eta'\boldsymbol{\beta}'\xi-\frac{1}{2\Sigma}(\xi'\boldsymbol{\Gamma}\xi+(\xi'\boldsymbol{\Gamma}'\xi)')$$

$$0=\frac{1}{\Sigma}\,\xi'\eta-\frac{1}{\Sigma}\,\eta'\boldsymbol{\beta}'\xi-\frac{1}{2\Sigma}\xi'\boldsymbol{\Gamma}\xi-\frac{1}{2\Sigma}\xi'\boldsymbol{\Gamma}\xi$$

$$0=\frac{1}{\Sigma}\,\xi'\eta-\frac{1}{\Sigma}\,\eta'\boldsymbol{\beta}'\xi-\frac{1}{\Sigma}\xi'\boldsymbol{\Gamma}\xi$$

$$\widehat{\boldsymbol{\gamma}}\left(\frac{1}{\Sigma}\,\xi'\xi\right)=\frac{1}{\Sigma}\,\xi'\eta-\frac{1}{\Sigma}\,\eta'\widehat{\boldsymbol{\beta}}'\xi$$

$$\widehat{\boldsymbol{\gamma}}= (\xi'\eta- \eta'\widehat{\boldsymbol{\beta}}'\xi)\,(\xi'\xi)^{-1}$$

For example, Y is a random variable with Y^T = (y_1, y_2, ..., y_p), because it is assumed to be normal then Y~N ($\boldsymbol{\Lambda}_y\eta$; $\boldsymbol{\Sigma}$). The probability density function is:

$$F(\Lambda_y\eta;\boldsymbol{\Sigma})=\frac{1}{\sqrt{(2\pi)^p|\boldsymbol{\Sigma}|}}\exp\left\{-\frac{1}{2\Sigma}\,(Y-\boldsymbol{\Lambda}_{\mathbf{y}}\eta)'(Y-\boldsymbol{\Lambda}_{\mathbf{y}}\eta)\right\}$$

If x = $\boldsymbol{\Lambda}_y\eta$ then the joint density function for random samples is stochastically independent and identical at x, as follows:

$$L= f(x_1), f(x_2), \ldots, f(x_n)$$

With likelihood function:

$$L(\theta)=\prod_{i=1}^{n} f(x_i)$$
$$=\ln\left\{(2\pi)^{\frac{-n(p)}{2}}|\boldsymbol{\Sigma}|^{-\frac{n}{2}}\exp\left\{-\frac{1}{2\Sigma}(Y-\boldsymbol{\Lambda}_{\mathbf{y}}\eta)'(Y-\boldsymbol{\Lambda}_{\mathbf{y}}\eta)\right\}\right\}$$
$$=\ln\left((2\pi)^{\frac{-n(p)}{2}}\right)+\ln\left(|\boldsymbol{\Sigma}|^{-\frac{n}{2}}\right)+\ln\exp\left\{-\frac{1}{2\Sigma}\left(Y'Y-2\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'Y+\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'\boldsymbol{\Lambda}_{\mathbf{y}}\eta\right)\right\}$$
$$=\frac{-n(p)}{2}\ln(2\pi)-\frac{n}{2}\ln(|\boldsymbol{\Sigma}|)-\frac{1}{2\Sigma}Y'Y+\frac{1}{\Sigma}\,\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'Y-\frac{1}{2\Sigma}\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'\boldsymbol{\Lambda}_{\mathbf{y}}\eta$$

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Estimation of parameter λ_y

$$0=\frac{\partial L(\theta)}{\partial \lambda_y}$$

$$=\frac{\partial\left(\frac{-n(p)}{2}\ln(2\pi)-\frac{n}{2}\ln(|\boldsymbol{\Sigma}|)-\frac{1}{2\boldsymbol{\Sigma}}Y'Y+\frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'Y-\frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'\boldsymbol{\Lambda}_{\mathbf{y}}\eta\right)}{\partial \lambda_y};\text{ with } \frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'\boldsymbol{\Lambda}_{\mathbf{y}}\eta$$

Based on the derivative properties of the matrix $\mathbf{f(x)=x'Ax}$ then $\frac{\partial f}{\partial \beta}=\mathbf{(A+A')x}$, so that the derivative of $\boldsymbol{\Lambda}_{\mathbf{y}}$ is obtained as follows:

$$0 = 0\text{-}\ 0\text{-}\ 0+\frac{1}{\boldsymbol{\Sigma}}\eta'Y-\frac{1}{2\boldsymbol{\Sigma}}(\eta'\boldsymbol{\Lambda}_{\mathbf{y}}\eta+(\eta'\boldsymbol{\Lambda}_{\mathbf{y}}'\eta)')$$

$$0=\frac{1}{\boldsymbol{\Sigma}}\eta'Y-\frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\Lambda}_{\mathbf{y}}\eta-\frac{1}{2\boldsymbol{\Sigma}}\eta'\boldsymbol{\Lambda}_{\mathbf{y}}\eta$$

$$0==\frac{1}{\boldsymbol{\Sigma}}\eta'Y-\frac{1}{\boldsymbol{\Sigma}}\eta'\boldsymbol{\Lambda}_{\mathbf{y}}\eta$$

$$\widehat{\boldsymbol{\Lambda}_{\mathbf{y}}}\left(\frac{1}{\boldsymbol{\Sigma}}\eta'\eta\right)=\frac{1}{\boldsymbol{\Sigma}}\eta'Y$$

$$\boldsymbol{\Lambda}_{\mathbf{y}}=\eta'Y(\eta'\eta)^{-1}$$

Let X be a random variable with $X^T = (x_1, x_2, \ldots, x_q)$, because it is assumed to be normal then $X \sim N(\boldsymbol{\Lambda}_x\xi; \boldsymbol{\Sigma})$. The probability density function is:

$$F(\boldsymbol{\Lambda}_x\xi;\boldsymbol{\Sigma})=\frac{1}{\sqrt{(2\pi)^q|\boldsymbol{\Sigma}|}}\exp\left\{-\frac{1}{2\boldsymbol{\Sigma}}(X-\boldsymbol{\Lambda}_{\mathbf{x}}\xi)'(X-\boldsymbol{\Lambda}_{\mathbf{x}}\xi)\right\}$$

If $x=\boldsymbol{\Lambda}_{\mathbf{x}}\xi$ then the joint density function for stochastically independent and identical random samples at x is as follows:

$$L=f(x_1),f(x_2),\ldots,f(x_n)$$

with the likelihood function:

$$L(\theta)=\prod_{i=1}^{n}f(x_i)$$

$$=\ln\left\{(2\pi)^{\frac{-n(q)}{2}}|\boldsymbol{\Sigma}|^{-\frac{n}{2}}\exp\left\{-\frac{1}{2\boldsymbol{\Sigma}}(X-\boldsymbol{\Lambda}_{\mathbf{x}}\xi)'(X-\boldsymbol{\Lambda}_{\mathbf{x}}\xi)\right\}\right\}$$

$$=\ln\left((2\pi)^{\frac{-n(q)}{2}}\right)+\ln\left(|\boldsymbol{\Sigma}|^{-\frac{n}{2}}\right)+\ln\exp\left\{-\frac{1}{2\boldsymbol{\Sigma}}(X'X-2\xi'\boldsymbol{\Lambda}_{\mathbf{x}}'X+\xi'\boldsymbol{\Lambda}_{\mathbf{x}}'\boldsymbol{\Lambda}_{\mathbf{x}}\xi)\right\}$$

$$=\frac{-n(q)}{2}\ln(2\pi)-\frac{n}{2}\ln(|\boldsymbol{\Sigma}|)-\frac{1}{2\boldsymbol{\Sigma}}X'X+\frac{1}{\boldsymbol{\Sigma}}\xi'\boldsymbol{\Lambda}_{\mathbf{x}}'X-\frac{1}{2\boldsymbol{\Sigma}}\xi'\boldsymbol{\Lambda}_{\mathbf{x}}'\boldsymbol{\Lambda}_{\mathbf{x}}\xi$$

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Estimation of parameter λ_x

$$0= \frac{\partial L(\theta)}{\partial \lambda_x}$$

$$= \frac{\partial\left(\frac{-n(p)}{2}\ln(2\pi) - \frac{n}{2}\ln(|\boldsymbol{\Sigma}|) - \frac{1}{2\boldsymbol{\Sigma}}X'X + \frac{1}{\boldsymbol{\Sigma}}\xi'\boldsymbol{\Lambda}_x{}'X - \frac{1}{2\boldsymbol{\Sigma}}\xi'\boldsymbol{\Lambda}_x{}'\boldsymbol{\Lambda}_x\xi\right)}{\partial \lambda_x}\text{; with } \frac{1}{2\boldsymbol{\Sigma}}\ \eta'\boldsymbol{\Lambda}_x{}'\boldsymbol{\Lambda}_x\xi$$

Based on the derivative properties of the matrix **f(x)= x′Λx** then $\frac{\partial f}{\partial \beta}$**= (A+A′)x**, so that the derivative of $\boldsymbol{\Lambda}_y$ is obtained as follows:

$$0 = 0\text{- }0\text{- }0+\frac{1}{\boldsymbol{\Sigma}}\ \xi'X\text{-}\frac{1}{2\boldsymbol{\Sigma}}\left(\xi'\boldsymbol{\Lambda}_x\xi + (\xi\boldsymbol{\Lambda}_x{}'\xi')'\right)$$

$$0 = \frac{1}{\boldsymbol{\Sigma}}\ \xi'X\text{-}\frac{1}{2\boldsymbol{\Sigma}}\xi'\boldsymbol{\Lambda}_x\xi\text{-}\frac{1}{2\boldsymbol{\Sigma}}\xi'\boldsymbol{\Lambda}_x\xi$$

$$0 = = \frac{1}{\boldsymbol{\Sigma}}\ \xi'X\ \text{-}\ \frac{1}{\boldsymbol{\Sigma}}\xi'\boldsymbol{\Lambda}_x\xi$$

$$\widehat{\boldsymbol{\Lambda}_x}\left(\frac{1}{\boldsymbol{\Sigma}}\ \xi'\xi\right) = \frac{1}{\boldsymbol{\Sigma}}\ \xi'X$$

$$\boldsymbol{\Lambda}_x\boldsymbol{=}\ \xi'X(\xi'\xi)^{-1}$$

In determining the estimated parameters γ, β and Γ using the maximum likelihood method, a closed form for the estimated parameters was not obtained so this was overcome using the help of Lisrel 8.80 software.

b. Maximum Likelihood Method Parameter Estimation with Lisrel 8.80

The results of the standardized solution and model estimation from Liserel 8.80 are as follows:

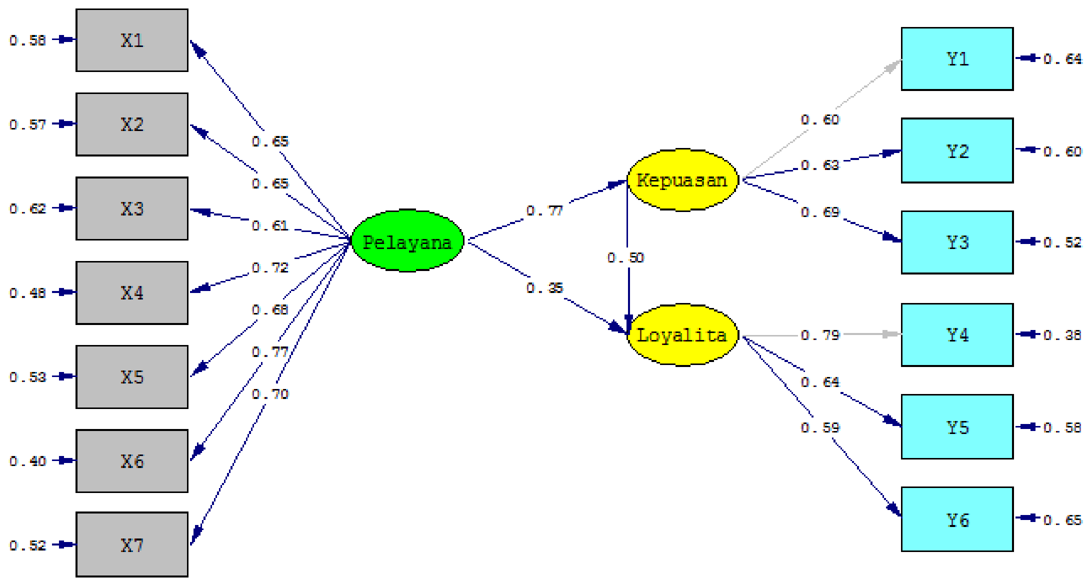

Figure 1. Standardized Solution Path Diagram.

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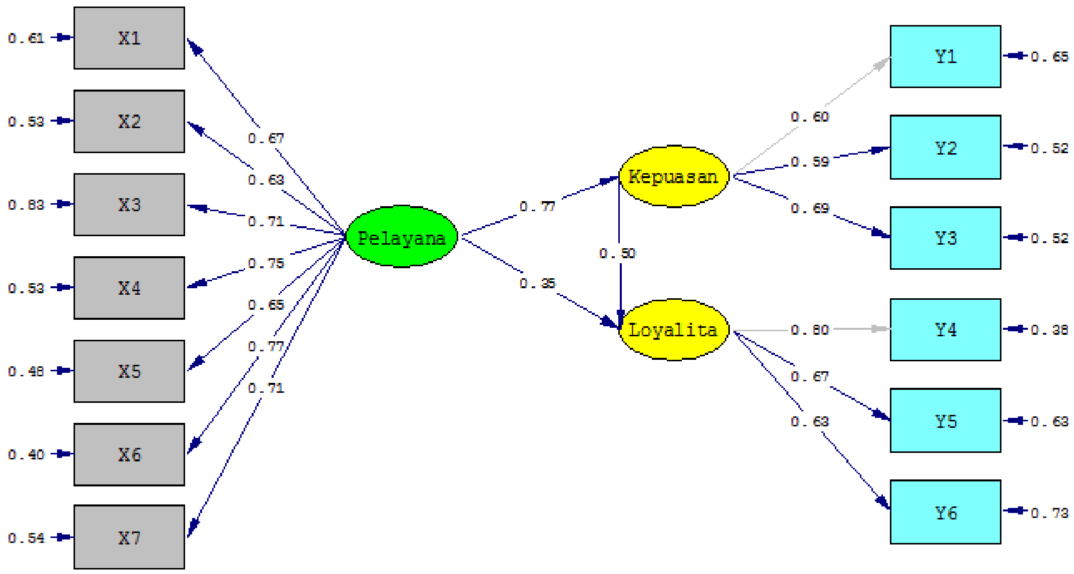

Figure 2. Path diagram of estimation results.

Based on Figures 1 and 2, it shows that the results of the standardized solution and estimates have the same close relationship value for each latent variable, namely from the service variable (ξ) to the satisfaction variable (η_1) of 0.77, the service variable (ξ) to loyalty (η_2) is 0.35 and the satisfaction variable (η_1) to the loyalty variable (η_2) is 0.50. Then the magnitude of the influence value on each indicator variable has almost the same value or not much different. With a sample size of 200, a chi-square value of 80.15 was obtained, where the smaller the chi-square value, the better and the chi-square value obtained can be considered quite good. The RMSEA value obtained is 0.038, which means close fit (not good), while for the p-value obtained, it is 0.06691, which is said to be good because the p-value is > 0.05, so it can be said that the data supports the desired model estimation.

The criteria for the significance test on the Standardized Loading Factor (SLF) and t-value are that the standard factor loading size is ≥ 0.5 and the t-value is ≥ 1.96, so it is said to be very significant or valid (Shrestha, 2021). To measure reliability, Construct Reliability (CR) ≥ 0.70 and Variance Extracted (VE) ≥ 0.50 are used (Elias et al., 2022; Fang et al., 2022). For more details, information regarding the validity and reliability of indicators in the measurement model can be seen in Table 1 below:

Table 1. Measurement Model Significance Test

Latent Variable	Indicator Variables	*Standardized Loading Factor*	*T-Value*	Information	Reliability		Information
					(VE ≥ 0.50)	(CR ≥ 0.70)	
	X1	0.65	9.78	Valid			Reliability
	X2	0.65	9.85	Valid	0.56	0.85	
Service	X3	0.61	9.11	Valid			
(ξ)	X4	0.72	11.16	Valid			

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	X5	0.68	10.44	Valid			
	X6	0.77	12.39	Valid			
	X7	0.70	10.70	Valid			
Satisfaction (η_1)	Y1	0.60	**	Valid	0.51	0.70	Reliability
	Y2	0.63	6.55	Valid			
	Y3	0.69	6.91	Valid			
Loyalty (η_2)	Y4	0.79	**	Valid	0.56	0.71	Reliability
	Y5	0.64	8.02	Valid			
	Y6	0.59	7.45	Valid			

Table 1 is a table of the results of calculating validity and reliability in the measurement model. The results show that all indicators on the latent variables have met the validity criteria with an SLF value for each indicator/construct of ≥ 0.5 with a t-value ≥ 1.96 (Radam et al., 2022). For each latent variable reliability of service, satisfaction and loyalty has a value of CR ≥ 0.70 and VE ≥ 0.5 so it can be said that the reliability of the latent variable service, satisfaction and loyalty in the measurement model is good. Apart from that, reliability can also be shown by the black standard error value on the path diagram, so it can be said that the indicator meets the reliability criteria in the measurement model.

c. Direct, Indirect, Total Influence
   To see the direct effect, see the picture:

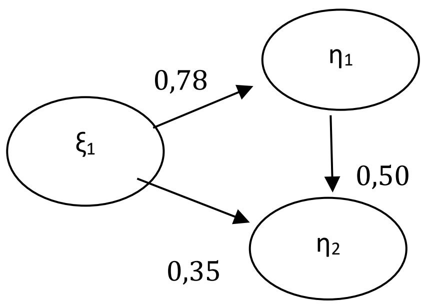

Figure 3. Direct Influence.

The value of the direct influence can be seen through the path coefficient value from one latent variable to another latent variable. The analysis results of the path coefficients are in Table 2 below:

Tabel 2. *Path Coefficients*

	Service (ξ_1)	Satisfaction (η_1)	Loyalty (η_2)
Service (ξ_1)	0	0.77	0.35
Satisfaction (η_1)	0	0	0.50
Loyalty (η_2)	0	0	0

From Table 2, it shows that the service path coefficient (ξ_1) on satisfaction (η_1) is 0.77, service (ξ_1) on loyalty (η_2) 0.35. So it can be concluded that the direct influence between

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the service variables (ξ_1) and satisfaction (η_1) is 0.77, service (ξ_1) on loyalty (η_2) is 0.35. This value also shows that service (ξ_1) has a positive effect on satisfaction (η_1) and loyalty (η_2).

The indirect effect is where service (ξ_1) significantly influences loyalty (η_2) through the intermediary variable satisfaction (η_1). The indirect influence can be seen in Figure 4:

$$\xi_1 \xrightarrow{0,77} \eta_1 \xrightarrow{0,50} \eta_2$$

Figure 4. Indirect Influence.

The indirect influence value is obtained by multiplying the path coefficients from service (ξ1) to satisfaction (η1) and from satisfaction (η1) to loyalty (η2). So the results obtained from multiplying the indirect influence path coefficient with the intermediate variable satisfaction (η1) are as follows:

Indirect influence through satisfaction (η_1)

$$\begin{aligned} \text{indirect influence} &= (\xi_1 \rightarrow \eta_1) \text{ x } (\eta_1 \rightarrow \eta_2) \\ &= (0.77) \text{ x } (0.50) \\ &= 0.385 \end{aligned}$$

Based on these results, it can be concluded that service (ξ_1) significantly influences loyalty (η_2) through the intermediary variable satisfaction (η_1) of 0.385. The total influence of service (ξ1) on loyalty (η2) with the intermediary variable satisfaction (η1) is 0.735, which means that the influence of service on customer loyalty through satisfaction is 73.5%.

d. Evaluation of the CB-SEM model
After analysis using the maximum likelihood method with Lisrel 8.80, an R-Square value was obtained which can state that service variability (ξ_1) explains 0.60 of customer satisfaction (η_1), which means that satisfaction variability (η_1) can be explained by 60% by service variability. (ξ_1) and service variability (ξ_1) explain 0.54 of the variability in customer loyalty (η_2), which means that variability in customer loyalty (η_2) can be explained by service variability (ξ_1) of only 54%.

CONCLUSION

Test the suitability of the entire model using goodness of fit criteria each value shows a good match, namely χ^2 (79.45) with the smaller the value, the better, NCP (17.45) with the smaller the value, the better, GFI (0.94) with the criterion value > 0.9, RMSEA (0.038) with a value measure of 0.05 ≤ good fit ≤ 0.08, AGFI (0.92) with a criterion value measure > 0.9, PNFI (0.77) with a value measure that increases the higher the better and NFI (0 .97) with a criterion value > 0.9. So it can be concluded that the overall model is good.The results of the analysis show that the influence of total service (ξ_1) on loyalty (η_2) with the intermediary variable satisfaction (η_1) is 0.735 which is This means that service has a big influence on customer loyalty through satisfaction namely 73.5%.

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