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Growth Model Study Using a Comparison of Gompertz, Logistic, and Weibull Models

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Abstract. Coronavirus Disease or COVID-19 has been a concern for the world, including Indonesia. The very rapid transmission of COVID-19 has had a wide impact on all communities around the world, especially Indonesia. To see the transmission of COVID-19 cases, which continues to increase rapidly, we can use a growth model. The growth model is a non-linear regression model that is used to describe growth behavior. These models can be exponential, sigmoidal, or S-shaped curves. The purpose of this study was to determine the growth curve model of positive COVID-19 cases in Indonesia using the Gompertz, Logistic, and Weibull models. After that, the model evaluation will be carried out using the coefficient of determination as a parameter, so that the best model will be obtained that can predict more accurately the growth of positive COVID-19 cases in Indonesia. The results obtained in this study are that the growth model for the Gompertz model is $g(x_i;\beta) = 8162350 \exp\{-e^{2.1306-0.0041x_i}\}$, the growth model for the Logistic model is $g(x_i;\beta) = \frac{6317713}{1+e^{4.6768-0.00880x_i}}$, and the growth model for the Weibull model is $g(x_i;\beta) = -454801 + 15145.6 \exp\{0.8262{x_i}^{0.2995}\}$. The best model that can predict the growth of positive COVID-19 cases in Indonesia is the Gompertz model, with a coefficient of determination is 0.99064.
Keywords: Gompertz Model; Growth Model; Logistic Model; Weibull Model

INTRODUCTION

Coronavirus Disease 2019 (COVID-19) is an infectious disease that has attracted global attention, including in Indonesia. This disease first emerged in December 2019 in Wuhan, China, and is caused by a new strain of the coronavirus family, the 2019 Novel Coronavirus (2019-nCoV), which was later officially named Severe Acute Respiratory Syndrome-Coronavirus 2 (SARS-CoV-2) (Bedford, et al., 2020). Coronaviruses themselves are a broad group of viruses that can infect both animals and humans. In humans, several types of coronavirus are known to cause respiratory infections, ranging from mild symptoms, such as the common cold, to serious illnesses, including Middle East Respiratory Syndrome (MERS) and Severe Acute Respiratory Syndrome (SARS). COVID-19 spreads primarily through droplets expelled when an infected individual coughs, sneezes, or speaks. Its high rate of transmission makes this virus a significant public health threat, with a wide impact across various sectors, including Indonesia (Tian, et al., 2020). The increasing spread of cases over time requires a quantitative analysis approach, one of which is through the application of growth models, to monitor and predict infection spread trends more accurately.

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The growth model is one of the non-linear regression models used to describe growth behavior because variables in growth tend to correlate with each other (Anindita, Wardhani, & Kusdarwati, 2014). Some of the most frequently used growth models are Brody, Richard, Logistic, von Bertalanffy, Weibull, and Gompertz. This growth model is often used because it is relatively easy in the calculation process and has a good ability to explain data in the field accurately, and can explain inflection points (Salman, et al., 2015).

This study employs three nonlinear growth models, namely, the Gompertz, Logistic, and Weibull, as predictive tools for analyzing the increasing trend of positive COVID-19 cases in Indonesia.The Gompertz, Logistic, and Weibull models have been widely used by researchers in various studies, such as human mortality studies conducted by Juckett & Rosenberg (1993), biological studies conducted by Kurnianto, Shinjo, & Suga (1998); Ismail, Khamis, & Jaafar (2003); and Mello et al. (2015), livestock studies conducted by Budimulyati et al. (2012); Sholikhin et al. (2016); Yusinta, Kurnianto, & Sutopo (2017); and Prayogo, Suprijatna, & Kurnianto (2017), and various other research case studies. In addition, many studies have been conducted on the study of COVID-19 growth, namely by Perez et al. (2020) and Asadi et al. (2020).

Referring to the previous description, this study aims to analyze the growth model of positive COVID-19 cases in Indonesia by comparing the Gompertz, Logistic, and Weibull models. The performance of each model is evaluated based on the coefficient of determination to identify the model with the best predictive accuracy.

METHOD

The data source used in this study is cumulative data on confirmed positive COVID-19 cases in Indonesia from March 1, 2020, to May 25, 2022, with a total of 816 daily observations. The data was obtained from the official COVID-19 Task Force portal (https://covid19.go.id/), which provides the latest and verified information on the development of the pandemic in Indonesia. The growth model analysis used in this study is a non-linear growth model, namely the Gompertz, Logistic, and Weibull models.

Nonlinear growth models are widely used to represent growth patterns that do not follow a straight line. These models are typically exponential or sigmoidal, such as an S-shaped curve, capable of depicting growth dynamics that experience acceleration and deceleration over time. The Gompertz model can be used as a growth curve with an S-shaped curve. Initially, the Gompertz model was used in agriculture and animal husbandry. In its development, the Gompertz model can be used in lifetime test models and can also be used to predict pandemic data. According to Piegorsch & Bailer (2005), the Gompertz growth model is as follows:

$$g(x_i;\beta) = \beta_0 \exp\{-\mathrm{e}^{-\beta_1-\beta_2 x_i)}. \quad (1)$$

One significant model that is often used to represent growth patterns over time is the Logistic growth model, which is particularly capable of capturing the dynamics of limited growth with a rate that slows down as it approaches maximum capacity. The Logistic growth model is as follows:

$$g(x_i;\beta) = \frac{\beta_0}{1+e^{-\beta_1-\beta_2 x_i}}. \quad (2)$$

Another sigmoid curve based on the exponential function is the Weibull growth model. This model adopts a different approach to relating unknown parameters to predictor variables, allowing flexibility in constructing various types of growth curves according to the characteristics of the data being analyzed. The Logistic growth model is as follows:

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$$g(x_i;\beta) = \beta_0 + \beta_1 \exp\{-\beta_2 {x_i}^{\beta_3}\}. \quad (3)$$

The iterative process in parameter estimation requires the calculation of partial derivatives for the estimated parameters. These partial derivatives, which will be used in the estimation algorithm, are presented in detail in Table 1 below.

Table 1. Partial Derivatives with respect to Each Parameter

Growth Model	Partial Derivative
Gompertz Model	$\frac{\partial g}{\partial \beta_0} = \exp\{-e^{-\beta_1-\beta_2 x}\}$ $\frac{\partial g}{\partial \beta_1} = \beta_0 \exp\{-\beta_1 - \beta_2 x - e^{-\beta_1-\beta_2 x}\}$ $\frac{\partial g}{\partial \beta_2} = x\beta_0 \exp\{-\beta_1 - \beta_2 x - e^{-\beta_1-\beta_2 x}\}$
Logistic Model	$\frac{\partial g}{\partial \beta_0} = (1 + e^{-\beta_1-\beta_2 x})^{-1}$ $\frac{\partial g}{\partial \beta_1} = \beta_0 e^{-\beta_1-\beta_2 x}(1 + e^{-\beta_1-\beta_2 x})^{-2}$ $\frac{\partial g}{\partial \beta_2} = x\beta_0 e^{-\beta_1-\beta_2 x}(1 + e^{-\beta_1-\beta_2 x})^{-2}$
Weibull Model	$\frac{\partial g}{\partial \beta_0} = 1$ $\frac{\partial g}{\partial \beta_1} = \exp\{-\beta_2 x^{\beta_3}\}$ $\frac{\partial g}{\partial \beta_2} = -x^{\beta_3}\beta_1 \exp\{-\beta_2 x^{\beta_3}\}$ $\frac{\partial g}{\partial \beta_3} = -x^{\beta_3}\log(x)\beta_1\beta_2 \exp\{-\beta_2 x^{\beta_3}\}$

The basic principle of the parameter estimation iteration process is to obtain the smallest residual squares from several combinations of the initial values determined, and these values must be based on previous research (Ismail, Khamis, & Jaafar, 2003). The process is stopped when it has reached a convergent condition. In this case, a computer program is needed to assist in estimating parameters in non-linear models. In this study, SAS 9.4 software was used to estimate parameters. SAS 9.4 software provides a special program to find non-linear parameters in the model using the NLIN (Non-Linear) procedure.

To obtain partial derivatives of parameters, this study will use the Marquardt method. The Marquardt model used in the iteration process requires partial derivatives of the estimated parameters. The partial derivatives of each non-linear growth model used can be seen in Table 1. Comparison of non-linear growth models is usually based on two criteria, namely ease of calculation and accuracy in depicting field data. The parameters used to evaluate these criteria are the number of iterations and the coefficient of determination (Salman, et al., 2015).

In this study, the parameter used to evaluate the model is the coefficient of determination. The coefficient of determination is a coefficient that describes the level of variation of field data that can be explained by a model (Salman, et al., 2015). The value of the coefficient of determination lies in the interval, and the model is said to be better if it approaches 1. The coefficient of determination value of each model is then compared to obtain the best model, so that the model obtained will be more accurate. The formula for the coefficient of determination is as follows (Chicco, Warens, & Jurman, 2021):

$$R^2 = \left(1 - \frac{RSS}{CTSS}\right), \quad (4)$$

where R^2 is the coefficient of determination (%), RSS is the Residual Sum Square, and CTSS is the Corrected Total Sum Square.

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RESULT AND DISCUSSION

In this study, parameter estimation was performed for each growth model Gompertz, Logistic, and Weibull to model the growth of positive COVID-19 cases in Indonesia. These models were then evaluated based on their coefficient of determination (R^2) to determine the most optimal model with the highest level of accuracy in predicting the development of positive COVID-19 cases in Indonesia.

After carrying out the iteration process on SAS 9.4, the results of the parameter estimates for the Gompertz growth model were obtained as $\beta_0 = 8162350$, $\beta_1 = -2.1306$, and $\beta_2 = 0.0041$, so that the Gompertz growth model for predicting the growth of positive COVID-19 cases in Indonesia is

$$g(x_i;\beta) = 8162350 \exp\{-\mathrm{e}^{2.1306-0.0041x_i)}\}.$$

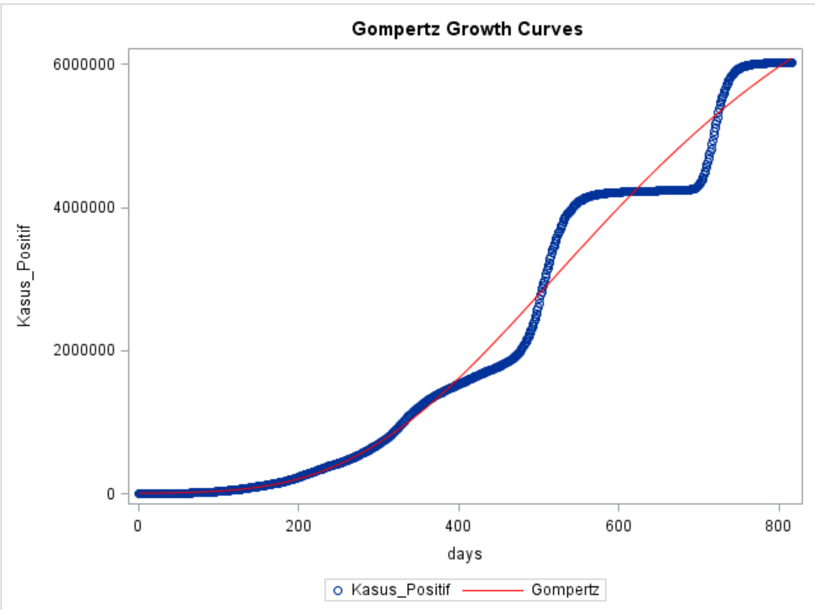
Then, for the Logistic growth model, the parameter estimates obtained were $\beta_0 = 6317713$, $\beta_1 = -4.6768$, and $\beta_2 = 0.00880$, so that the Logistic growth model for the growth of positive COVID-19 cases in Indonesia is

$$g(x_i;\beta) = \frac{6317713}{1+e^{4.6768-0.0088x_i}}.$$

In addition, for the Weibull growth model, the parameter estimates obtained were $\beta_0 = -454801$, $\beta_1 = 15145.6$, $\beta_2 = -0.8262$, and $\beta_3 = 0.2995$, so that the Logistic growth model for the growth of positive COVID-19 cases in Indonesia is

$$g(x_i;\beta) = -454801 + 15145.6 \exp\{0.8262{x_i}^{0.2995}\}.$$

Based on the growth model obtained, a comparison of the growth curve of COVID-19 cases in Indonesia for each model can be seen in Figure 1 below.



(a)

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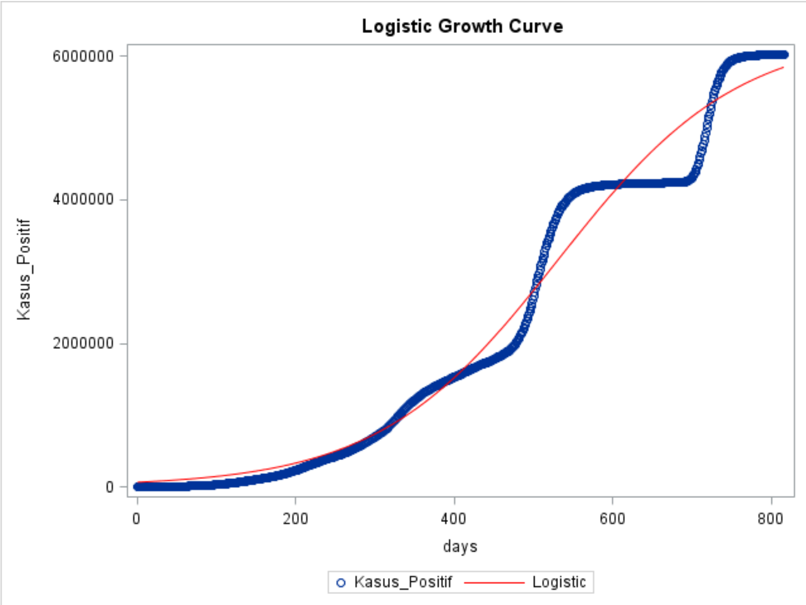
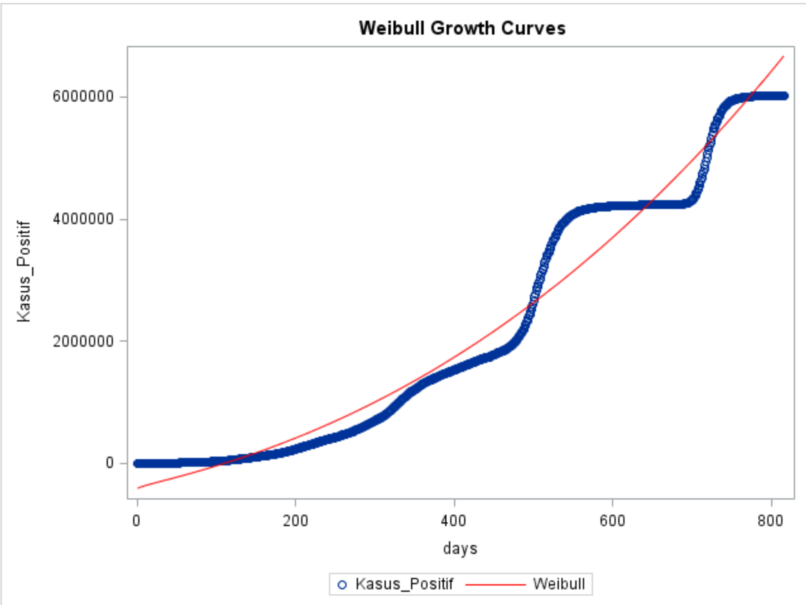


(b)



(c)

Figure 1. Growth curves of the (a) Gompertz, (b) Logistic, and (c) Weibull models.

Figure 1 above shows the growth curve of positive COVID-19 cases in Indonesia. The blue dots show the actual data of positive COVID-19 cases in Indonesia, while the red line shows the data of suspected positive COVID-19 cases in Indonesia based on the Gompertz, Logistic, and Weibull models.

In this study, the parameter used to evaluate the accuracy of the model in predicting the growth of positive COVID-19 cases in Indonesia is the coefficient of determination. If the coefficient of determination value is close to 1, then it can be said that the model can predict

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the growth model more accurately. For the calculation of the coefficient of determination value in each model, it has been presented in the following table.

Table 2. Comparison of Determination Coefficient Values

Growth Model	R^2 Value
Model Gompertz	0.99064
Model Logistik	0.98961
Model Weibull	0.96834

Based on Table 2 above, the coefficient of determination value that is closer to 1 is the Gompertz growth model, which is 0.99064, when compared to the Logistic growth model of 0.98961 and the Weibull growth model of 0.96834. Therefore, the best model that can predict the growth of positive COVID-19 cases in Indonesia more accurately is the Gompertz model.

CONCLUSION

Based on the results and discussions in the previous chapter, it is concluded that the growth model for the Gompertz model is $g(x_i;\beta) = 8162350 \exp\{-e^{2.1306-0.0041x_i)}\}$, the growth model for the Logistic model is $g(x_i;\beta) = \frac{6317713}{1+e^{4.6768-0.00880x_i}}$, and the growth model for the Weibull model is $g(x_i;\beta) = -454801 + 15145.6 \exp\{0.8262{x_i}^{0.2995}\}$. With the best model that can predict more accurately the growth of positive COVID-19 cases in Indonesia, namely the Gompertz model with a coefficient of determination value of 0.99064.

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